# Discussion papers about FPMC2021-68739

On occasion of the ASME/BATH Symposium on Fluid Power and Motion Control (FPMC 2021) a paper has been published written by Christian Schänzle and Peter Pelz from the Technical University of Darmstadt in Germany:

C. Schänzle, P. Pelz, “Meaningful and physically consistent efficiency definition for positive displacement pumps - continuation of the critical review of ISO 4391 and ISO 4409” in Proceedings of 2019 ASME/BATH Symposium on Fluid Power and Motion Control. American Society of Mechanical Engineers, 2021.

The paper can be regarded as a continuation of a number of articles, in particular:

- P. Achten, R. Mommers, T. Nishiumi, H. Murrenhoff, N. Sepehri, K. Stelson, J.-O. Palmberg, and K. Schmitz, “Measuring the losses of hydrostatic pumps and motors: A critical review of ISO4409:2007”, in Proceedings of 2019 ASME/BATH Symposium on Fluid Power and Motion Control. American Society of Mechanical Engineers, 2019, https://doi.org/10.1115/FPMC2019-1615

- P.Y. Li and J.H. Barkei, “Hydraulic effort and the efficiencies of pump and motors with compressible fluid”, in Proceedings of 2020 ASME/BATH Symposium on Fluid Power and Motion Control. American Society of Mechanical Engineers, 2020, https://doi.org/10.1115/FPMC2020-2801

Preceding the conference, a written discussion developed as a result of the review process, having the two authors (Christian Schänzle and Peter Pelz) on the one side, and the ‘reviewers’ (Peter Achten and Robin Mommers) on the other side. At the end of this discussion, it was suggested by Christian Schänzle and Peter Pelz to make the written documents of this discussion open for the public domain. This was als agreed in the conference during the open discussion of the paper.

This document is a compilation of the 7 documents:

1. First manuscript of Christian Schänzle and Peter Pelz: “Meaningful Efficiency Definition for All Positive Displacement Pumps - Continuation of the Critical Review of ISO 4391 and 4409” (downloaded on May 11, 2021)
2. First review of paper FPMC2021-68739 by Peter Achten and Robin Mommers  (send on May 21, 2021 to the organizers of the FPMC)
3. Final version of the paper from Christian Schänzle and Peter Pelz as accepted by the organizers of the FPMC. Paper_210805_FPMC21_Meaningful_Efficiency_final_schaenzle_pelz (received on August 5, 2021)
4. First rebuttal of Christian Schänzle and Peter Pelz to the first review of Peter Achten and Robin Mommers Note_210805_Response_Review_AchtenEtAl (first rebuttal, received on August 5, 2021)
5. Comment on mass density (send by Robin Mommers on August 6, 2021 to the authors)
6. Response by Peter Achten and Robin Mommers to the first rebuttal: ’note_210805_Response_Review_AchtenEtAl’ (send on August 13, 2021 to the organizers of the FPMC and to the authors)
7. Response from Christian Schänzle and Peter Pelz to the previous document Note_210818_Response_Review_Achten_Pelz_schaenzle (received on August 21, 2021)
8. Comment from Robin Mommers and Peter Achten to Note_210818 (Comment_210825_note_Pelz, send on August 25, 2021)

All the above mentioned documents are attached below.

1. Meaningful Efficiency Definition for All Positive Displacement Pumps - Continuation of the Critical Review of ISO 4391 and 4409 (downloaded on May 11, 2021)

**Proceedings of the ASME/BATH 2021**
**Symposium on Fluid Power and Motion Control**
**FPMC2021**
**October 19-21, 2021, Virtual, Online**

# FPMC2021-XXXX

# MEANINGFUL AND PHYSICALLY CONSISTENT EFFICIENCY DEFINITION FOR POSITIVE DISPLACEMENT PUMPS - CONTINUATION OF THE CRITICAL REVIEW OF ISO 4391 AND ISO 4409

**Christian Schänzle**
Chair of Fluid Systems
Technische Universität Darmstadt
Darmstadt, Germany
Email: christian.schaenzle@fst.tu-darmstadt.de

**Peter F. Pelz**
Chair of Fluid Systems
Technische Universität Darmstadt
Darmstadt, Germany
Email: peter.pelz@fst.tu-darmstadt.de

## ABSTRACT

*ISO 4391:1984 gives the common efficiency definition for positive displacement machines. ISO 4409:2019 uses this efficiency definition to specify the procedure for efficiency measurements. If the machine conditions do not correspond with an incompressible flow due to operation at high pressure levels, the compressibility of the fluid and the dead volume of a pump must be taken into account. On this point, ISO 4391:1984 is physically inconsistent.*

*Achten et. al. address this issue in their paper at FPMC 2019 presenting a critical review of ISO 4409:2007. They introduce new definitions of the overall efficiency as well as the mechanical-hydraulic efficiency. At the same time, they question the validity of the volumetric efficiency definition. Li and Barkei continue on this issue in their paper at FPMC 2020 and give a new efficiency definition based on the introduction of a new quantity $\Phi$ which describes the volume specific enthalpy of the conveyed fluid.*

*The motivation of this paper is to contribute to the ongoing and fruitful discussion. Our approach starts with the most general efficiency definition, namely the isentropic efficiency. Subsequently, we make assumptions concerning the fluid properties with respect to the compressibility of the conveyed fluid. On the basis of the ideal cycle of a positive displacement pump and the p-v diagram, we derive physically consistent and more meaningful representations of the overall, the mechanical-hydraulic and the volumetric efficiency that address the inconsistency of ISO 4391:1984. Furthermore, we compare our findings with the existing results of Achten et. al. and Li and Barkei.*

## NOMENCLATURE

| | |
|---|---|
| $e$ | mass-specific internal energy |
| $g$ | gravitational body force |
| $h_\mathrm{l}$ | loss enthalpy |
| $\Delta h_\mathrm{s}$ | mass-specific isentropic enthalpy difference |
| $\Delta h_\mathrm{t,s}$ | total mass-specific and isentropic enthalpy difference |
| $\Delta H_\mathrm{s}$ | isentropic enthalpy difference |
| $M_\mathrm{hyd}$ | hydraulic torque |
| $M_\mathrm{mh}$ | friction torque |
| $M_\mathrm{S}$ | shaft torque |
| $m$ | mass |
| $m_\mathrm{eff}$ | effective mass |
| $m_\mathrm{F}$ | mass of conveyed fluid |
| $\dot{m}$ | mass flow |
| $n$ | rotational speed |
| $P_\mathrm{loss}$ | power loss |
| $\dot{Q}$ | heat flow |
| $Q_\mathrm{eff}$ | effective volume flow |
| $Q_\mathrm{L}$ | leakage |
| $V$ | experimentally determined displacement volume |
| $V_\mathrm{eff}$ | effective displacement volume |
| $V_\mathrm{t}$ | total volume |
| $V_\mathrm{d}$ | dead volume |
| $p$ | pressure |
| $P_\mathrm{S}$ | shaft power |
| $s$ | entropy |
| $u$ | mean velocity at inlet or outlet of a machine |
| $\kappa$ | isentropic compressibility |

| | |
|---|---|
| $K_S$ | isentropic bulk modulus |
| $\eta$ | isentropic efficiency |
| $\eta_{mh}$ | mechanical-hydraulic efficiency |
| $\eta_{vol}$ | volumetric efficiency |
| $\varrho$ | density |
| $\Phi$ | volume-specific enthalpy |

## 1. INTRODUCTION

The common efficiency definition for positive displacement machines is given by ISO 4391:1984 [1]. However, the overall efficiency definition and the definitions of the partial efficiencies, namely the volumetric efficiency and mechanical-hydraulic efficiency are physically consistent only for an incompressible flow with the density $\varrho = \text{const.}$ If the machine operates at high pressure levels the compressibility of the fluid and the dead volume of a pump must be taken into account. On this point, ISO 4391:1984 is physically inconsistent.

Achten et. al. [2] address this issue in their paper at FPMC 2019 presenting a critical review of ISO 4409:2007 [3]. ISO 4409:2007 specifies the procedure of efficiency measurements and adopts the efficiency definition from ISO 4391:1984. Meanwhile, a new version of ISO 4409 from 2019 exists (ISO 4409:2019 [4]), that no longer explicitly states efficiency definitions and instead only refers to ISO 4391:1984. Consequently, a critical review of efficiency definitions must address ISO 4391:1984. Achten et. al. introduce a new definition of the overall efficiency as well as the mechanical-hydraulic efficiency discussing the influence of the compressibility of a fluid as well as the dead volume of a positive displacement machine. At the same time, they question the validity of a volumetric efficiency definition.

Li and Barkei [5] continue on this issue in their paper at FPMC 2020 and also give new definitions of the overall and partial efficiencies considering a compressible flow. They introduce a new quantity $\Phi$ which designates the volume specific enthalpy of the conveyed fluid and serves as the equivalent to $\Delta p$ considering the efficiency definitions for an incompressible flow. Furthermore, their approach makes no assumptions regarding compressibility or the relation between pressure and density. Based on a comparison of their own efficiency definition with the definition of Achten et. al and Williamson and Manrig [6], Li and Barkei still find differences and inconsistencies among the results.

The motivation of this paper is to contribute to the ongoing discussion and to give a meaningful and physically consistent representation for the overall, the mechanical-hydraulic and the volumetric efficiency. At the beginning in section 2, we give the most general efficiency definition which is the starting point of our considerations. In the following, we make assumptions concerning the fluid properties which are analogous to the ones made by Achten et. al., namely the linearization of the constitutive relation between pressure and density and the use of an averaged isentropic bulk modulus. In section 3, on the basis of the assumptions, the ideal cycle of a positive displacement pump and the $p$-$V$ diagram, we derive a physically consistent representation of the converted energy in a positive displacement machine. Both extensive and intensive quantities are considered. Consequently, we obtain physically consistent and meaningful representations of the overall efficiency, the volumetric efficiency and the mechanical-hydraulic efficiency. Section 4 compares the results of this paper with the definitions of Achten et. al. and Li and Barkei. Finally, section 5 gives the conclusion.

## 2. ENERGY BALANCE AND ASSUMPTIONS FOR EFFICIENCY DEFINITIONS

The assessment of the energy conversion in a positive displacement machine is based on the assumption that the machine operates stationary on a time-averaged basis. Hence, the first law of thermodynamics reads

$$P_S + \dot{Q} = \dot{m}\Delta h_t, \qquad (1)$$

with the mass flow $\dot{m}$, the difference of the mass-specific total enthalpy between machine outlet and inlet $\Delta h_t$, the mechanical shaft power $P_S = 2\pi M_S n$ being the product of the shaft torque $M_S$ and the rotational speed $n$, and the heat flow $\dot{Q}$. All quantities are considered to be averaged over time.

In the case of a pump, $P_S$ and $\Delta h_t$ are both greater than zero, in the case of a motor, $P_S$ and $\Delta h_t$ are negative. The mass flow at the inlet and outlet of a machine are identical. In case of an external leakage $\dot{m}_L$ it is assumed to be redirected to the inlet of the pump or the outlet of a motor respectively as shown in FIGURE 1. Due to environmental constraints, real external leakage is unlikely.

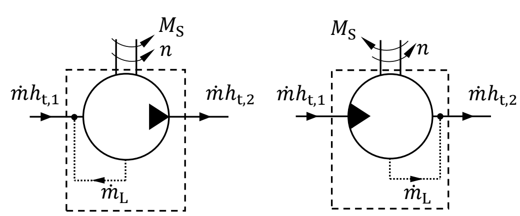


**FIGURE 1**: FIRST LAW OF THERMODYNAMICS FOR AN ADIABATIC POSITIVE DISPLACEMENT PUMP AND MOTOR.

The commonly used efficiency definition for positive displacement machines is the isentropic efficiency. In fact, this efficiency definition is used for all machines, turbo machines or positive displacement machines with a compressible flow or an incompressible flow, as long as the machine operates adiabatically. Considering an adiabatic machine, the isentropic efficiency $\eta$ is defined as the ratio of the product of mass flow and mass-specific isentropic and total enthalpy difference $\Delta h_{t,s}$ and the shaft power

$$\eta := \left(\frac{\dot{m}\Delta h_{t,s}}{P_S}\right)^{\pm 1}. \qquad (2)$$

The exponent $+1$ applies to pumps, the exponent $-1$ applies to motors. While the following sections focus on pumps, the described procedure can be applied on motors similarly.

Dividing the mass-specific total enthalpy difference $\Delta h_{\mathrm{t}}$ into the isentropic fraction $\Delta h_{\mathrm{t},s}$ and the loss fraction $h_{\mathrm{l}}$, we obtain the following representation of the isentropic efficiency

$$\eta := 1 - \frac{\dot{m} h_{\mathrm{l}}}{P_{\mathrm{S}}}. \tag{3}$$

Equation (3) illustrates that the efficiency is a measure of the dissipative power losses $P_{\mathrm{loss}} = \dot{m} h_{\mathrm{l}}$.

Given an approximately incompressible flow and an ideally rigid machine, the total enthalpy $h_{\mathrm{t}}$ is

$$\begin{aligned} h_{\mathrm{t}} &= \frac{p}{\varrho} + \frac{u^2}{2} + gz + e, \\ \varrho h_{\mathrm{t}} &= p + \varrho \frac{u^2}{2} + \varrho g z + \varrho e = \Delta p_{\mathrm{t}} + \varrho e \end{aligned} \tag{4}$$

equation (2) leads to the ISO 4391:1984 efficiency definition

$$\eta := \frac{\Delta p_{\mathrm{t}} Q}{P_{\mathrm{S}}} = \frac{\Delta p_{\mathrm{t}} Q}{2\pi M_{\mathrm{S}} n}, \tag{5}$$

with the total pressure difference $\Delta p_{\mathrm{t}}$ and the volume flow $Q$. Extending equation (5) with the displacement volume $V$, the efficiency can be written as the product of the volumetic efficiency $\eta_{\mathrm{vol}}$ and the mechanical-hydraulic efficiency $\eta_{\mathrm{mh}}$

$$\eta = \eta_{\mathrm{vol}} \eta_{\mathrm{mh}}, \qquad \eta_{\mathrm{vol}} := \frac{Q}{nV}, \qquad \eta_{\mathrm{mh}} := \frac{\Delta p_{\mathrm{t}} V}{2\pi M_{\mathrm{S}}}. \tag{6}$$

The displacement volume needs to be determined experimentally on the basis of Toet's method [7].

In the case of high pressure differences, the mass-specific isentropic internal energy difference $\Delta e_s$ must not be neglected which represents the converted energy due to compression. Hence, the compressibility of the fluid needs to be taken into account. At this point, we make the following two assumptions:

(i) The compression and decompression of the fluid is isentropic ($s = \mathrm{const}$) and can be described using an averaged isentropic bulk moduls $\bar{K}_{\mathrm{S}}$ or averaged isentropic compressibility $\bar{\kappa} = 1/\bar{K}_{\mathrm{S}}$.

(ii) The relation between volume $V$ and pressure $p$ of a fluid

$$\bar{\kappa} := -\frac{1}{V}\frac{\mathrm{d}V}{\mathrm{d}p}\Big|_s \tag{7}$$

is linearized and yields

$$\bar{\kappa} \approx -\frac{1}{V}\frac{\Delta V}{\Delta p}\Big|_s. \tag{8}$$

As can be seen in section 3.2, these assumptions are not mandatory but can be easily extended by pressure dependent compressibility $\kappa(p)$. However, the assumptions shorten the efficiency representations derived from the isentropic efficiency definition in equation (2), as can be seen in the next section. Furthermore, Ivantysyn und Ivantysynova [8] state that the resulting error due to linearization for common hydraulic fluids is negligible.

## 3. EFFICIENCY DEFINTION BASED ON THE IDEAL CYCLE WITH DEAD VOLUME

Based on the above assumptions, the next step is to determine the numerator of the isentropic efficiency definition in equation (2), which is the product of mass flow and the difference of the mass-specific isentropic enthalpy $\Delta h_{\mathrm{s}}$ (kinetic energy $u^2/2$ and potential energy $gz$ are neglected).

### 3.1 Ideal cycle based on extensive quantities

Firstly and for reasons of clarity, we focus on the isentropic enthalpy difference $\Delta H_{\mathrm{s}}$ of the conveyed fluid mass per cycle, which is an extensive quantity. The isentropic enthalpy difference is equivalent to the energy transferred between machine and fluid per rotation. The time averaged mass flow

$$\dot{m} := \frac{1}{T}\int_0^T \tilde{\dot{m}}(t)\,\mathrm{d}t \tag{9}$$

is given by the time integral of the temporal mass flow $\tilde{\dot{m}}(t)$ and the cycle time $T = 1/n$. Hence, the time averaged mass flow is the product of conveyed fluid mass per rotation $m_{\mathrm{F}}$ and the rotational speed $n$

$$\dot{m} = n m_{\mathrm{F}}. \tag{10}$$

This yields

$$\dot{m}\Delta h_{\mathrm{s}} = n \Delta H_{\mathrm{s}}. \tag{11}$$

FIGURE 2 shows the ideal cycle of a positive displacement pump with a dead volume $V_{\mathrm{d}}$ filled by a compressible fluid in a $p$-$V$-diagram. The shaded area, given by the points abcd, states the isentropic enthalpy difference $\Delta H_{\mathrm{s}}$ and needs to be calculated in order to derive a meaningful efficiency representation based on the efficiency definition (2) and equation (11). The dead volume results from the design of a positive displacement pump and must be calculated on the basis of the geometric pump dimensions. The displacement volume $V$ is determined experimentally at a pressure difference $\Delta p = 0$ (cf. [7]).

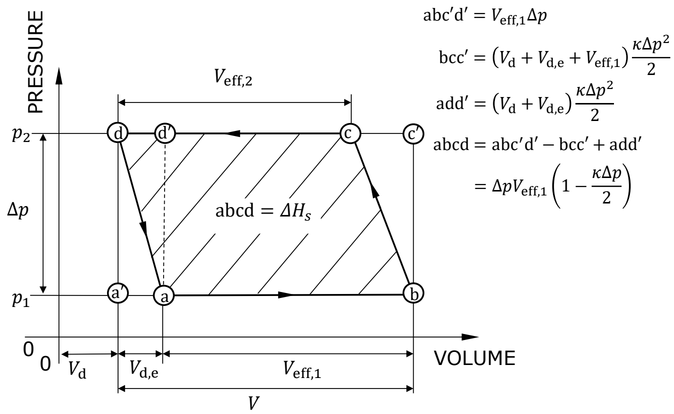


**FIGURE 2**: IDEAL CYCLE FOR A POSITIVE DISPLACEMENT PUMP WITH A DEAD VOLUME AND A COMPRESSIBLE FLUID.

Beginning the ideal cycle at the top dead center, point d, and the pressure $p_2$, the dead volume decompresses, d→a, before refilling the displacement chamber at the pressure level $p_1$, d→a. The difference between the compressed and expanded dead volume is called $V_{\mathrm{d,e}}$. In the following, the fluid with the effective displacement volume $V_{\mathrm{eff},1}$ flows into the displacement chamber, a→b. At the bottom dead center, point b, the total volume $V_{\mathrm{t}}$ of the displacement chamber is

$$V_{\mathrm{t}} = V_{\mathrm{d}} + V = V_{\mathrm{d}} + V_{\mathrm{d,e}} + V_{\mathrm{eff},1}. \tag{12}$$

The effective displacement volume $V_{\mathrm{eff},1}$ is given by

$$V_{\mathrm{eff},1} = V - V_{\mathrm{d,e}} \tag{13}$$

and equation (8) yields

$$V_{\mathrm{d,e}} = V_{\mathrm{d}}\bar{\kappa}|\Delta p|\,. \tag{14}$$

Consequently, the effective displacement volume $V_{\mathrm{eff},1}$ can be calculated from the experimentally determined displacement volume $V$, the geometrically calculated dead volume $V_{\mathrm{d}}$, the averaged isentropic compressibility $\bar{\kappa}$ and the pressure difference $\Delta p$. This is of major importance, as the decompression of the dead volume and reduction of usable displacement volume $\Delta V = V_{\mathrm{d,e}} = V - V_{\mathrm{eff},1}$ does not cause volumetric losses or dissipation of energy. Instead, it underlines the effective Volume $V_{\mathrm{eff},1}$ being the relevant geometric quantity in the partial efficiencies, the volumetric and mechanical-hydraulic efficiency.

Further on, the total volume is compressed to the pressure level $p_2$, b→c, and displaced from of the displacement chamber until the top dead center is reached again, c→d.

The isentropic enthalpy difference $\Delta H_{\mathrm{s}}$ (abcd) can now be calculated from the following areas, each described by its corner points

$$\mathrm{abcd} = \mathrm{abc'd'} - \mathrm{bcc'} + \mathrm{add'}. \tag{15}$$

Each area can be easily calculated based on the edge lengths. These correspond to the pressure difference $\Delta p$, the effective displacement volume $V_{\mathrm{eff},1}$ and the volume difference due to compression, b→c, or expansion, d→a, calculated with equation (8). The results for the different areas are

$$\begin{aligned} \mathrm{abc'd'} &= \Delta \mathrm{p} V_{\mathrm{eff},1}\,, \\ \mathrm{bcc'} &= \left(\mathrm{V_d} + \mathrm{V_{d,e}} + V_{\mathrm{eff},1}\right)\frac{\bar{\kappa}\Delta p^2}{2}\,, \\ \mathrm{add'} &= \left(\mathrm{V_d} + \mathrm{V_{d,e}}\right)\frac{\bar{\kappa}\Delta p^2}{2}. \end{aligned} \tag{16}$$

The area $\mathrm{add'} = \mathrm{ada'}$ is calculated from the perspective of compressing the decompressed dead volume $V_{\mathrm{d}} + V_{\mathrm{d,e}}$, a→d. In this way, the compression energy can be represented by $\mathrm{V_d} + \mathrm{V_{d,e}}$. Since the compression or decompression is assumed to be isentropic, the absolute value of the converted mechanical energy is equal, a→d = d→a. However, due to the assumption made, namely the linearized relation in equation (8), there is a deviation between expansion and compression:

$$\begin{matrix}\text{area add'} \\ \text{expansion (d→a)}\end{matrix} \quad = \frac{V_{\mathrm{d}}\bar{\kappa}\Delta p^2}{2}, \tag{17}$$

$$\begin{matrix}\text{area add'} \\ \text{compression (a→d)}\end{matrix} \quad = \frac{\left(V_{\mathrm{d}} + V_{\mathrm{d,e}}\right)\bar{\kappa}\Delta p^2}{2}. \tag{18}$$

This deviation results from the linearization error which is negligible for the range of practical pressures and therefore not considered any further. Equations (15) and (16) now leads to the isentropic enthalpy difference $\Delta H_{\mathrm{S}}$ of the conveyed fluid mass $m_{\mathrm{eff}} = V_{\mathrm{eff},1}\varrho_1$ per rotation

$$\mathrm{abcd} = \Delta p V_{\mathrm{eff},1}\left(1 - \frac{\bar{\kappa}\Delta p}{2}\right). \tag{19}$$

Equation (19) gives a short, meaningful and physically consistent representation of the isentropic enthalpy difference which can be used for the efficiency representation based on the definition in equation (2). Before that, we derive the same result based on the mass specific isentropic enthalpy, which is an intensive quantity.

 

## 3.2 Ideal cycle based on intensive quantities

Due to internal leakage, the measured mass flow rate $\dot{m} = nm_\mathrm{F}$ (cf. eq. (10)) will differ from the effective mass flow $nm_\mathrm{e} = \varrho_1 V_{\mathrm{eff},1} n$ with the density $\varrho_1 = \varrho(p_1)$. It is therefore advantageous to use the mass-specific isentropic enthalpy difference $\Delta h_\mathrm{s}$, which is an intensive quantity:

$$\Delta h_\mathrm{s} = \frac{\Delta H_\mathrm{s,e}}{m_\mathrm{eff}} = \frac{\Delta p}{\varrho_1}\left(1 - \frac{\bar{\kappa}\Delta p}{2}\right). \tag{20}$$

Equation (20) can also be derived from the mass-specific $p$-$v$-diagram shown in FIGURE 3. The mass-specific isentropic enthalpy difference from state 1 to 2 also results in

$$\Delta h_s = \int_1^2 v \mathrm{d}p \approx \frac{\Delta p}{\varrho_1}\left(1 - \frac{\bar{\kappa}\Delta p}{2}\right). \tag{21}$$

One obtains the isentropic enthalpy change $\Delta H_\mathrm{s}$ (abcd) by multiplying the corresponding fluid masses of the dead volume $m_\mathrm{d}$ and the effectively conveyed volume $m_\mathrm{eff} = \varrho_1 V_{\mathrm{eff},1}$

$$\begin{aligned} \mathrm{abcd} = \Delta H_\mathrm{s} &= (m_\mathrm{eff} + m_\mathrm{d})\Delta h_\mathrm{s} - m_\mathrm{d}\Delta h_\mathrm{s} \\ &= m_\mathrm{eff}\Delta h_\mathrm{s} \\ &= \Delta p V_{\mathrm{eff},1}\left(1 - \frac{\bar{\kappa}\Delta p}{2}\right). \end{aligned} \tag{22}$$

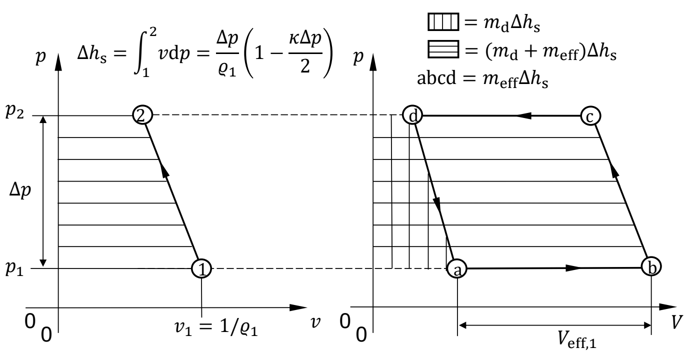


**FIGURE 3:** MASS-SPECIFIC $p$-$v$-DIAGRAM FOR AN IDEAL POSITIVE DISPLACEMENT PUMP AND A COMPRESSIBLE FLUID.

At the same time, it is obvious that the mass-specific isentropic enthalpy difference based on equation (21) can also be calculated with a non-linearized relationship of pressure and density (cf. equation (7)) and a pressure dependent compressibility $\kappa(p)$.

## 3.3 Efficiency representations

Following the efficiency definition according to definition (2) and equation (21) one obtains

$$\eta := \frac{\dot{m}\Delta h_\mathrm{s}}{2\pi M_\mathrm{S} n} = \frac{Q_1 \Delta p}{2\pi M_\mathrm{S} n}\left(1 - \frac{\bar{\kappa}\Delta p}{2}\right). \tag{23}$$

The mass flow $\dot{m} = \varrho_1 Q_1$ is the product of the volume flow $Q_1$ and the density $\varrho_1$ at the pump inlet. Since the volume flow is usually measured at the pump outlet, the volume flow $Q_1$ can be calculated with equation (8) by

$$Q_1 = \frac{Q_2}{1 - \bar{\kappa}\Delta p}. \tag{24}$$

Based on equation (23), representations of the partial efficiencies can be derived, which also give a physically consistent and meaningful measure for the volumetric and mechanical-hydraulic losses:

(i) The leakage $Q_\mathrm{L} = Q_{\mathrm{eff},1} - Q_1$ represents the difference between the effective volume flow $Q_{\mathrm{eff},1} = nV_{\mathrm{eff},1}$ and the measured volume flow at pump inlet $Q_1$. The leakage causes the power loss $P_\mathrm{loss,L} = \Delta p Q_\mathrm{L}$.

(ii) The friction torque $M_\mathrm{mh} = M_\mathrm{S} - M_\mathrm{hyd}$ is calculated from the difference of shaft torque $M_\mathrm{S}$ and hydraulic torque $M_\mathrm{hyd} = \frac{\Delta p V_{\mathrm{eff},1}}{2\pi}\left(1 - \frac{\bar{\kappa}\Delta p}{2}\right)$. This results in the power loss $P_\mathrm{loss,mh} = 2\pi M_\mathrm{mh} n$.

Extending equation (23) with the effective displacement volume $V_{\mathrm{eff},1}$ (cf. equation (13) and (14)) in the numerator and denominator, the isentropic efficiency yields

$$\eta := \frac{Q_1}{nV_{\mathrm{eff},1}} \frac{\Delta p V_{\mathrm{eff},1}}{2\pi M_\mathrm{S}}\left(1 - \frac{\bar{\kappa}\Delta p}{2}\right). \tag{25}$$

Consequently, definitions of the volumetric efficiency $\eta_\mathrm{vol}$ and the mechanical-hydraulic efficiency $\eta_\mathrm{mh}$ can be given by

$$\begin{aligned} \eta_\mathrm{vol} &:= \frac{Q_1}{nV_{\mathrm{eff},1}} = 1 - \frac{Q_\mathrm{L}}{nV_{\mathrm{eff},1}}, \\ \eta_\mathrm{mh} &:= \frac{\Delta H_\mathrm{S}}{2\pi M_\mathrm{S}} = \frac{\Delta p V_{\mathrm{eff},1}}{2\pi M_\mathrm{S}}\left(1 - \frac{\bar{\kappa}\Delta p}{2}\right) \\ &= \frac{1}{1 + \frac{2\pi}{1 - \bar{\kappa}\,\Delta p/2}\frac{M_\mathrm{mh}}{\Delta p V_\mathrm{e}}} \end{aligned} \tag{26}$$

Equations (23) and (26) provide representations of the overall efficiency, the volumetric and the mechanical-hydraulic efficiency which measure the energetic quality of positive displacement pumps with a dead volume in a physically consistent and meaningful way. Hence, the energetic quality can also be quantified based on the volumetric losses $Q_\mathrm{L}$ and the friction or momentum losses of the conveyed fluid $M_{\mathrm{mh}}$. If the dead volume is negligibly small or the flow is approximately incompressible due to low pressure differences of the pump, the effective displacement volume $V_{\mathrm{eff,1}}$ and the experimentally determined displacement volume $V$ will be identical. Furthermore, $\kappa \Delta p \ll 1$ and the efficiency definitions of ISO 4391 according to equations (5) and (6) can be applied.

## 4. COMPARISON OF EFFICIENCY REPRESENTATIONS IN THE LITERATURE

As stated in the introduction the motivation of this paper is to contribute to the ongoing and fruitful discussion about meaningful and physically consistent efficiency representations of positive displacement machines with a dead volume and a compressible flow which was started at FPMC 2019 by Achten et. al [2]. Against this background, the overall efficiency and partial efficiency representations derived in this paper are compared to the efficiency representations given by Achten et. al. and Li and Barkei [4]. TABLE 1 summarizes all efficiency representations in the notation of this paper considering a pump with one single displacement chamber.

Achten et. al. make new proposals for the overall and the mechanical hydraulic efficiency. They also calculate the isentropic enthalpy $\Delta H_{\mathrm{S}}$ from the ideal cycle of a positive displacement machine (cf. FIGURE 2) but derive a more extensive formula resulting in a more extensive representation of the mechanical efficiency as well. This is due to the calculation of the area add′ (cf. FIGURE 2) from the perspective of an expansion (see equations (17) and (18)) and due to the linearization error. On the other hand, Achten et. al.'s definition of the overall efficiency is physically inconsistent. They integrate the inner energy (see [2] equation (5)) neglecting the pressure dependent density $\varrho$. Consequently, this leads to a physically inconsistent result of the hydraulic power as well which is the nominator of the overall efficiency. At the same time, they question the validity of a volumetric efficiency definition and, thus, do not provide one. A physically consistent volumetric efficiency definition that fulfills $\eta = \eta_{\mathrm{mh}}\eta_{\mathrm{vol}}$, is not achievable due to their overall efficiency definition. Furthermore, their view on the volumetric efficiency does not address rotating positive displacement pumps, which (i) are used at lower pressures, (ii) usually have no or a negligible dead volume, and for which (iii) volumetric losses are often decisive for efficiency. In this regard, the volumetric efficiency must be a measure of the power losses due to leakage.

Li and Barkei give generally valid and physically consistent definitions of the overall efficiency, the volumetric and the mechanical hydraulic efficiency. These definitions contain their newly introduced quantity $\Phi$ which is the volume-specific enthalpy (cf. FIGURE 2)

$$\Phi := \frac{\Delta H_{\mathrm{S}}}{V_{\mathrm{eff,2}}}. \tag{27}$$

In this way, they do not make any assumptions regarding the fluid properties, namely the compressibility of the fluid, e.g. by using an averaged bulk modulus or by linearizing the relation of pressure and density. However, this is why their approach results in efficiency representations that are slightly more difficult to understand. Regardless of this, Li and Barkei's representations are identical to the representations derived in this paper when taking into account the assumptions made in section 2.

## 5. CONCLUSION

On the basis of the most general efficiency definition, namely the isentropic efficiency, the assumptions considering the fluid properties and the $p$-$v$ diagram, we derive physically consistent and meaningful representations of the overall, the volumetric and the mechanical-hydraulic efficiency. These representations are consistent with the definitions of Li and Barkei [4] and may serve as a template for a revision of ISO 4391:1984 [1]. In particular, the use of the effective volume $V_{\mathrm{eff,1}}$ at a low-pressure level (cf. equation (13) and (14)) is the basis of a short and comprehensible efficiency representation.

**TABLE 1**:EFFICIENCY DEFINITIONS OF PUMPS.

| | |
|---|---|
| **volumetric efficiency** | |
| Achten et. al: | no definition |
| Li and Barkei: | $\eta_{\mathrm{vol}} := \dfrac{Q_1}{nV_{\mathrm{eff,1}}} = \dfrac{Q_2}{nV_{\mathrm{eff,2}}}$ |
| this paper: | $\eta_{\mathrm{vol}} := \dfrac{Q_1}{nV_{\mathrm{eff,1}}} = \dfrac{Q_2}{nV_{\mathrm{eff,2}}}$ |
| **mechanical-hydraulic efficiency** | |
| Achten et. al: | $\eta_{\mathrm{mh}} := \dfrac{\Delta p V}{2\pi M_{\mathrm{S}}}\left[1 - \Delta p \bar{\kappa}\left(\dfrac{1}{2} + \dfrac{V_{\mathrm{d}}}{V}\right)\right]$ |
| Li and Barkei: | $\eta_{\mathrm{mh}} := \dfrac{V_{\mathrm{eff,2}}\Phi(\Delta p, p_1)}{2\pi M_{\mathrm{S}}}$ <br> $\Phi(\Delta p, p_1) := \dfrac{1}{V_{\mathrm{eff,2}}}\displaystyle\int_1^2 V \mathrm{d}p = \dfrac{\Delta H_{\mathrm{S}}}{V_{\mathrm{eff,2}}}$ |
| this paper: | $\eta_{\mathrm{mh}} := \dfrac{\Delta p V_{\mathrm{eff,1}}}{2\pi M_{\mathrm{S}}}\left(1 - \dfrac{\bar{\kappa}\Delta p}{2}\right) = \dfrac{\Delta H_{\mathrm{S}}}{2\pi M_{\mathrm{S}}}$ <br> $= \dfrac{\Delta p V_{\mathrm{eff,2}}}{2\pi M_{\mathrm{S}}}\dfrac{(1 - \bar{\kappa}\Delta p/2)}{(1 - \bar{\kappa}\Delta p)}$ |

 

| overall efficiency | |
|---|---|
| Achten et. al: | $\eta := \frac{\mathrm{p}_2 Q_2 \left(1 + \frac{p_2 \bar{\kappa}}{2}\right) - p_1 Q_1}{2\pi M_S n}$ |
| Li and Barkei: | $\eta := \frac{Q_2 \Phi(\Delta p, p_1)}{2\pi M_S n}$ |
| this paper: | $\eta \coloneqq \frac{\Delta p Q_1}{2\pi M_S n}\left(1 - \frac{\bar{\kappa}\Delta p}{2}\right) = \frac{\Delta p Q_2}{2\pi M_S n}\frac{(1 - \bar{\kappa}\Delta p/2)}{(1 - \bar{\kappa}\Delta p)}$ |

## REFERENCES


[1] ISO 4391:1984, “Hydraulic fluid power; pumps, motors and integral transmissions, parameter definitions and letter symbols”, Beuth Verlag, Berlin, Germany, 1984.

[2] P. Achten, R. Mommers, T. Nishiumi, H. Murrenhoff, N. Sepehri, K. Stelson, J.-O. Palmberg, and K. Schmitz, “Measuring the losses of hydrostatic pumps and motors: A critical review of Iso4409:2007,” in *Proceedings of 2019 ASME/BATH Symposium on Fluid Power and Motion Control*. American Society of Mechanical Engineers, 2019.

[3] ISO 4409:2007, “Hydraulic fluid power – Positive displacement pumps, motors and integral transmissions – Methods of testing and presenting basic steady state performance”, Beuth Verlag, Berlin, Germany, 2007.

[4] ISO 4409:2019, “Hydraulic fluid power – Positive displacement pumps, motors and integral transmissions – Methods of testing and presenting basic steady state performance”, Beuth Verlag, Berlin, Germany, 2007.

[5] P.Y. Li and J.H. Barkei, “Hydraulic effort and the efficiencies of pump and motors with compressible fluid” in *Proceedings of 2020 ASME/BATH Symposium on Fluid Power and Motion Control*. American Society of Mechanical Engineers, 2020.

[6] C. Williamson and N. Manring, “A more accurate definition of mechanical and volumetric efficiencies for digital displacement pumps” in *Proceedings of 2019 ASME/BATH Symposium on Fluid Power and Motion Control*. American Society of Mechanical Engineers, 2019.

[7] G. Toet, J. Johnson, J. Montague, K. Torres and J. Garcia-Bravo, „The Determination of the Theoretical Stroke Volume of Hydrostatic Positive Displacement Pumps and Motors from Volumetric Measurements“, in *Energies 12.3*, 2019.

[8] J. Ivantysyn und M. Ivantysynova, "Hydrostatische Pumpen und Motoren: Konstruktion und Berechnung“, Vogel, Würzburg, Germany, 1993.

2. First review of paper FPMC2021-68739 by Peter Achten and Robin Mommers (send on May 21, 2021 to the organizers of the FPMC)

**Review**

# Meaningful and Physically Consistent Efficiency Definition for Positive Displacement Pumps - Continuation of the Critical Review of ISO 4391 and 4409 (FPMC2021-68739)

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## Introduction

Contrary to normal conventions we (Peter Achten and Robin Mommers) decided, as reviewers, not to have an anonymous review, but instead to have an open (ongoing and fruitful) discussion. Obviously we consider the review of ISO4409 and 4391 an important discussion and we highly value your thoughts and concerns.

The intend of our 2019-paper[1] was to propose a new set of definitions and equations for the losses and efficiencies of hydrostatic pumps and motors. The new set was meant as a replacement of the equations which were at that time defined in ISO4409:2007.

It was our goal to create a new set of efficiency and loss definitions which are:

- Practical, i.e. not too complicated for general use;
- Useful, which means that you can measure or determine the parameters involved with sufficient accuracy;
- Generic, i.e. valid for all hydrostatic pumps and motors, including variable displacement pumps and motors. But also including units with a zero or near zero dead volume, or units which are used at lower pressures, or units which predominantly have high volumetric losses.

The new equations are not a precise representation of the physical reality (if ever this is possible), but sufficiently accurate to have a better understanding of the losses, than current conventions and standards allow us. There are many effects and influences that we have considered to be of less importance and for which we have decided that these effects could be ignored or neglected. Many of these assumptions have been discussed with our colleagues in academia and industry, also outside the group of authors of our 2019-paper.

We would also encourage you to read our report 'Performance of Hydrostatic Machines' from INNAS, which can be downloaded at the INNAS website (www.innas.com). Annex B of this report discusses a sensitivity analysis of the bulk modulus and the possible effects of a pressure dependent density model.

To quote this report: "In conclusion, it is found that the alleged increase in accuracy gained by using a detailed oil density model to determine the bulk modulus is probably negligible compared to the accuracy of the measurement results. For the sake of clarity as well as simplicity, the use of a constant value for the isentropic bulk modulus during the calculation of hydrostatic performances will suffice."

We have chosen to assume a constant density and bulk modulus model (as you do as well, at least implicitly in your analysis) because the influence is rather small and would result in much more complicated definitions of all efficiencies and losses.

[1] Achten, P., R. Mommers, T. Nishiumi, H. Murrenhoff, N. Sepehri, K. Stelson, J.-O. Palmberg, K. Schmitz, 'Measuring the losses of hydrostatic pumps and motors - A critical review of ISO 4409, Proc. FPMC2019, ASME/Bath Symposium on Fluid Power and Motion Control, October 7-9, 2019, Sarasota, Florida, USA (FPMC2019-1615)

# Consensus

Let us first start with the consensus: we both agree upon the necessity for a revision of the ISO-standards with respect to definition and equations for the overall efficiency, the hydro-mechanical and the volumetric efficiency of hydrostatic pumps and motors. We furthermore agree that the effects of the bulk modulus of the oil need to be included in the loss analysis of hydrostatic pumps and motors. Although the equations, which are proposed by you in Table 1, don't mention explicitly any influence of the dead volume, there is an implicit influence through the relation between $V$ and $V_{eff,1}$. Substituting Eq.14 in Eq.13 results in:

$$V_{eff,1} = V - V_d \, \bar{\kappa} \left| \Delta p \right| \tag{1}$$

In this equation, $V_d$ represents the dead volume. In our paper, this parameter is called $V_{min}$. Consequently, we both agree that the influence of the dead volume should be included in the efficiency definitions.

At first sight it seems that your equations for the mechanical-hydraulic efficiency and the overall efficiency differ much from ours, but in reality they are almost the same. Also here we (almost) agree. That can be explained as follows:

Your paper results in the following definition of the mechanical-hydraulic efficiency:

$$\eta_{mh} = \frac{\Delta p V_{eff,1}}{2\pi M_S} \left( 1 - \frac{\bar{\kappa} \Delta p}{2} \right) \tag{2}$$

Substituting the above equation (Eq.1) for $V_{eff,1}$ results in:

$$\eta_{mh} = \frac{\Delta p V}{2\pi M_S} \left( 1 - \bar{\kappa} \Delta p \left[ \frac{1}{2} + \frac{V_d}{V} \right] + \frac{V_d}{V} \frac{\left[ \bar{\kappa} \Delta p \right]^2}{2} \right) \tag{3}$$

This is almost the same equation as ours (see Table 1 in your manuscript), except for the term:

$$\frac{V_d}{V} \frac{\left[ \bar{\kappa} \Delta p \right]^2}{2}$$

Just to give you an idea of the magnitude, we can calculate the value of:

$$\left( 1 - \bar{\kappa} \Delta p \left[ \frac{1}{2} + \frac{V_d}{V} \right] \right) \tag{4}$$

being our correction factor, and

$$\left( 1 - \bar{\kappa} \Delta p \left[ \frac{1}{2} + \frac{V_d}{V} \right] + \frac{V_d}{V} \frac{\left[ \bar{\kappa} \Delta p \right]^2}{2} \right) \tag{5}$$

being the correction factor that follows from your equation. Assuming:

$$
\begin{aligned}
&\bar{\kappa} = 6\mathrm{E}-10\ [\mathrm{Pa}^{-1}] \\
&\Delta p = 400\ [\mathrm{bar}] = 4E7\ [\mathrm{Pa}] \\
&V_d/V = 0.7\ [\text{-}]
\end{aligned}
\tag{6}
$$

our correction factor becomes a value of 0.9713 and yours 0.9715, a difference of 0.0002.

Furthermore your equation for the overall efficiency can be rewritten as follows:

$$
\begin{aligned}
\eta &= \frac{\Delta p Q_2}{2\pi M_S}\frac{\left(1-\bar{\kappa}\Delta p/2\right)}{\left(1-\bar{\kappa}\Delta p\right)} = \frac{\Delta p Q_2}{2\pi M_S}\frac{\left(1+\bar{\kappa}\Delta p/2-\bar{\kappa}\Delta p\right)}{\left(1-\bar{\kappa}\Delta p\right)} = \\
&= \frac{\Delta p Q_2}{2\pi M_S}\left(1+\frac{\bar{\kappa}\Delta p/2}{\left(1-\bar{\kappa}\Delta p\right)}\right)
\end{aligned}
\tag{7}
$$

This is rather similar to our equation:

$$
\eta = \frac{p_2 Q_2\left(1+\bar{\kappa}\Delta p/2\right) - p_1 Q_1}{2\pi M_S}
\tag{8}
$$

If, for the moment, we ignore the fact that we split $p_2$ and $p_1$, whereas you consider the pressure difference $\Delta p$, than our correction term is:

$$
\left(1+\bar{\kappa}\Delta p/2\right)
\tag{9}
$$

whereas yours is:

$$
\left(1+\frac{\bar{\kappa}\Delta p/2}{\left(1-\bar{\kappa}\Delta p\right)}\right)
\tag{10}
$$

Again using the parameters mentioned in Eq(6), our correction factor has a value of 1,0120 and yours of 1,0123, a difference of 0,0003.

These differences are so small that we can conclude that we largely have a consensus about the new definitions for the mechanical-hydraulic and the overall efficiency.

---

## Differences

Aside from the consensus, we also have some important differences:

1. As mentioned in the introduction, our equations are for <u>all</u> hydrostatic pumps and motors, also pumps and motors with an external case drain or a pre-charged oil supply (i.e. $p_1 \neq 0$). You consider the outgoing mass flow equal to the input, which means you exclude all pumps and motors having an external case drain. Furthermore:

   $$p_2 Q_2 - p_1 Q_1 \neq \Delta p Q \tag{11}$$

   We believe that you are making a mistake when assuming that $\Delta pQ = p_2Q_2 - p_1Q_1$ (as you do in your paper) Not only when $p_1 \neq 0$, but also because a pressurised and heated up flow is different from a low pressure flow at another temperature. This is also the reason why your definition of the overall efficiency differs from ours. It should be noted that also ISO4391 mentions a separation of the $p_2$ and $p_1$ flows.

2. Whenever the pressure level at the low pressure side is higher than the case pressure, it is no longer certain from which pressure level the volumetric losses come from. Part of the loss will come from the high pressure side of the pump, but another part will come from the low pressure side. Since the bearing gaps are often larger at the low pressure side, the leakage from the low pressure side is often considerable, despite the lower pressure level.

3. This argument seems to be of no concern if the pump or motor housing has the same pressure level as the low pressure side of the unit, but even then the Δp at which leakage occurs is uncertain. The thermodynamic cycle, represented in the *pV*-diagram, is not a closed cycle. You may use it for determining the indicated work of a single cycle (which can be used for calculating the hydro-mechanical efficiency), but you can't use it for calculating the volumetric efficiency.

   As an example, consider a hydrostatic axial piston motor taking high pressure oil from a high pressure supply line. During commutation the high and low pressure side are connected via the silencing grooves. At that moment there is a short circuit connection, and the motor is just taking oil from the high pressure line as much as is needed. However, during the commutation, the pressure in the commutating cylinder increases. As a result, the short circuit leakage flow occurs at variable pressure differentials. Consequently your assumption in section 3.3, point (i) that $P_{\text{loss,L}} = \Delta p Q_L$ is incorrect.

4. We object to the idea that the volumetric efficiency can be defined as $Q_1/nV_{\text{eff},1}$ (Eq.26 and Table 1 in your manuscript). This is a flow ratio, not an energy or power ratio. Whereas the denominator could be multiplied by a pressure level (in order to convert it to a power unit), this can't be done with the numerator, since you don't know from which pressure level or pressure differential the leakage flow originates. This is also the reason why we didn't define a volumetric efficiency.

5. In section 4 you write that we "question the validity of a volumetric efficiency definition and, thus, do not provide one. A physically consistent volumetric efficiency definition that fulfils $\eta = \eta_{\text{mh}}\,\eta_{\text{vol}}$, is not achievable due to their overall efficiency definition."

   We didn't provide a definition of the volumetric efficiency because we couldn't find a definition which was physically consistent with the inner processes in hydrostatic pumps and motors. You can make and define a flow ratio, but that is not the same as a power or energy ratio (which was the topic of our paper).

The fact that we didn't come up with a definition of the volumetric efficiency has nothing to do with our definition of the overall efficiency. After all, in theory, it could be possible to make an equation in which our overall efficiency is divided by our definition of the hydro-mechanical efficiency, which would then result in a 'volumetric efficiency' which fulfils $\eta = \eta_{mh}\, \eta_{vol.}$ But this would not make any sense due to the reasons mentioned before.

6. The difference between your equation for the mechanical-hydraulic efficiency and ours is due to the calculation of the area **abcd** in your paper, which is the equivalent of $E_i$ in our paper. You are correct that the linearisation of Eq.7 in your paper to Eq.8 results in an error. We also agree that this error is negligible. However, due to the linearisation, the area add' can result in slightly different equations (as you indicate in Eqs.17 and 18). The difference is however negligible (as you have mentioned yourself). Nevertheless, you chose to use Eq.18 for your further analysis. This is inconsistent with Eq.14 in which you define $V_{d,e}$. It should be clear that the triangular area add' can be calculated as:

$$\mathrm{add}' = \tfrac{1}{2}\Delta p V_{d,e} \tag{12}$$

When substituting your Eq.14 from the manuscript in the equation above, you'll get:

$$\mathrm{add}' = \frac{V_{d,e}\bar{\kappa}\Delta p}{2} \tag{13}$$

which equals Eq. 17 in your manuscript. If you would have continued to use this equation to calculate the area **abcd**, then your definition of the hydro-mechanical efficiency would result in the same equation as ours, aside from the difference in using Δp (which you do), versus splitting the energy levels $p_2$ and $p_1$ (like we do). It should also be noted that also for the calculation of the area **bcc',** the same difference between 'expansion' and 'compression' could be made as you did for **add'.** Also in that case, the choice would be rather arbitrary and only result in a negligible error. It is however remarkable that you noted the difference for **add'** but not for the calculation of area **bcc'**. Furthermore, the use of Eq.17 does not correspond to your definition of $V_{d,e}$ (Eq. 14).

---

## Comments

1. How do you suggest that $V_{eff,1}$ can be measured?

2. If $V_{eff,1}$ needs to be calculated from Eq.13 from your manuscript, wouldn't it be better to define your equations also based on $V$ instead of $V_{eff,1}$?

3. We would have preferred if, instead of Eq.1 in your manuscript you would have used our first step in the analysis:

$$\begin{aligned} P_S+\dot{Q} &= \dot{m}_2 h_2-\dot{m}_1 h_1 = \\ &= Q_2\rho_2\left(u_2+\frac{p_2}{\rho_2}\right)-Q_1\rho_1\left(u_1+\frac{p_1}{\rho_1}\right)= \\ &= \left(Q_2\rho_2 u_2-Q_1\rho_1 u_1\right)+\left(p_2 Q_2-p_1 Q_1\right) \end{aligned} \tag{14}$$

   Aside from being a more general equation, the above equation also clarifies that, in our opinion, you can't just multiply Δp with Q, as you do in Eq.23

4. In Eq.4 you suddenly convert the local pressure $p$ to a pressure differential $\Delta p_t$ (for which no definition is given in the nomenclature). We believe this is wrong.

5. The area **add'** can also be calculated on the basis of the volume $V_{d,e}$, which would then result in Eq. 17 to be used for further analysis.

6. Do you agree that the differences between your definition of the overall efficiency and ours is very small if not negligible?

7. Do you agree that the differences between your definition of the mechanical-hydraulic efficiency and ours is very small if not negligible?

8. Do you agree that you would have the same definition for the mechanical-hydraulic efficiency if you would have used Eq.17 for calculating **abcd**?

9. It is not possible to define the volumetric power loss $P_{loss,L}$ as $\Delta p Q_L$.

10. It is not possible to make a definition for the volumetric efficiency in terms of a power or energy ratio.

11. It is incorrect that our definition of the overall efficiency in 'physically inconsistent'. We urgently invite you to show us why or where you see any physical inconsistency.

12. We disagree that we have a different overall and a different hydro-mechanical efficiency because of the linearisation error. We both make the same linearisation error. We both agree that this error is negligible.

13. We indeed assume the density to be constant in Eq.5 of our paper (your remark in section 4). However your analysis and equations are not different from ours in this perspective. The error of this assumption has been quantified in our test report and can be considered to be negligible.

14. In section 4 you write that we can't make a physically consistent volumetric efficiency definition due to our overall efficiency definition. We kindly ask you to explain this statement.

15. In section 4 you write that our view on the volumetric efficiency does not address rotating positive displacement pumps which are used at lower pressures. We kindly ask you to explain this statement.

16. In section 4 you write that our view on the volumetric efficiency does not address rotating positive displacement pumps which usually have no or a negligible dead volume. We kindly ask you to explain this statement. Please note that in our report 'Performance of Hydrostatic Machines' we also show the test results of a Marzocchi pump which has a zero dead volume.

17. In section 4 you write that our view on the volumetric efficiency does not address rotating positive displacement pumps for which volumetric losses are often decisive. We kindly ask you to explain this statement.

18. In the conclusion you emphasise the importance of basing your equations on $V_{\mathrm{eff},1}$. As mentioned before, we can't understand how this results in a practical set of equations since there is no way of measuring $V_{\mathrm{eff},1}$

19. TABLE 1: The volumetric efficiency from your manuscript (and that of Li and Barkei) are wrong if considered from an energy analysis point of view.

20. TABLE 1: Your equation for the mechanical-hydraulic efficiency is almost equal to ours. The differences are due to a different assumption in the linearisation of the commutation processes. The differences are in the end negligible.

21. TABLE 1: As a general formula for pumps and motors it is wrong to assume to use $\Delta p$ for a single flow ($Q_1$ or $Q_2$) instead of a separation of the high and low pressure flows.

22. Finally: Why is it so important that the overall efficiency needs to be the product of a hydro-mechanical efficiency and some kind of volumetric efficiency? Energy efficiencies can be multiplied in order to get an overall efficiency, if <u>and only if</u> the processes are in series. An example is the combination of an electric motor and a pump, for which you may multiply the efficiency of the electric motor with the efficiency of the pump. But the friction and leakage losses in a pump are not processes which can be separated, nor can they be considered to run in series.

May, 2021
Peter Achten and Robin Mommers
INNAS

3. Final version of the paper from Christian Schänzle and Peter Pelz as accepted by the organizers of the FPMC.
Paper_210805_FPMC21_Meaningful_Efficiency_final_schaenzle_pelz (received on August 5, 2021)

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# FPMC2021-XXXX

# MEANINGFUL AND PHYSICALLY CONSISTENT EFFICIENCY DEFINITION FOR POSITIVE DISPLACEMENT PUMPS - CONTINUATION OF THE CRITICAL REVIEW OF ISO 4391 AND ISO 4409

**Christian Schänzle**
Chair of Fluid Systems
Technische Universität Darmstadt
Darmstadt, Germany
Email: christian.schaenzle@fst.tu-darmstadt.de

**Peter F. Pelz**
Chair of Fluid Systems
Technische Universität Darmstadt
Darmstadt, Germany
Email: peter.pelz@fst.tu-darmstadt.de

## ABSTRACT

*ISO 4391:1984 gives the common efficiency definition for positive displacement machines. ISO 4409:2019 uses this efficiency definition to specify the procedure for efficiency measurements. If the machine conditions do not correspond with an incompressible flow due to operation at high pressure levels, the compressibility of the fluid and the dead volume of a pump must be taken into account. On this point, ISO 4391:1984 is physically inconsistent.*

*Achten et. al. address this issue in their paper at FPMC 2019 presenting a critical review of ISO 4409:2007. They introduce new definitions of the overall efficiency as well as the mechanical-hydraulic efficiency. At the same time, they question the validity of the volumetric efficiency definition. Li and Barkei continue on this issue in their paper at FPMC 2020 and give a new efficiency definition based on the introduction of a new quantity $\Phi$ which describes the volume specific enthalpy of the conveyed fluid.*

*The motivation of this paper is to contribute to the ongoing and fruitful discussion. Our approach starts with the most general efficiency definition, namely the isentropic efficiency. Subsequently, we make assumptions concerning the fluid properties with respect to the compressibility of the conveyed fluid. On the basis of the ideal cycle of a positive displacement pump and the p-v diagram, we derive physically consistent and more meaningful representations of the overall, the mechanical-hydraulic and the volumetric efficiency that address the inconsistency of ISO 4391:1984. Furthermore, we compare our findings with the existing results of Achten et. al. and Li and Barkei.*

## NOMENCLATURE

| | |
|---|---|
| $e$ | mass-specific internal energy |
| $g$ | gravitational body force |
| $h_\mathrm{l}$ | loss enthalpy |
| $\Delta h_\mathrm{s}$ | mass-specific isentropic enthalpy difference |
| $\Delta h_\mathrm{t,s}$ | total mass-specific and isentropic enthalpy difference |
| $\Delta H_\mathrm{s}$ | isentropic enthalpy difference |
| $M_\mathrm{hyd}$ | hydraulic torque |
| $M_\mathrm{mh}$ | friction torque |
| $M_\mathrm{S}$ | shaft torque |
| $m$ | mass |
| $m_\mathrm{eff}$ | effective mass |
| $m_\mathrm{F}$ | mass of conveyed fluid |
| $\dot{m}$ | mass flow |
| $n$ | rotational speed |
| $P_\mathrm{loss}$ | power loss |
| $\dot{Q}$ | heat flow |
| $Q_\mathrm{eff}$ | effective volume flow |
| $Q_\mathrm{L}$ | leakage |
| $V$ | experimentally determined displacement volume |
| $V_\mathrm{eff}$ | effective displacement volume |
| $V_\mathrm{L}$ | leakage volume per rotation |
| $V_\mathrm{t}$ | total volume |
| $V_\mathrm{d}$ | dead volume |
| $p$ | pressure |
| $P_\mathrm{S}$ | shaft power |
| $s$ | entropy |
| $u$ | mean velocity at inlet or outlet of a machine |
| $\kappa$ | isentropic compressibility |

| | |
|---|---|
| $K_S$ | isentropic bulk modulus |
| $\eta$ | isentropic efficiency |
| $\eta_{\mathrm{mh}}$ | mechanical-hydraulic efficiency |
| $\eta_{\mathrm{vol}}$ | volumetric efficiency |
| $\varrho$ | density |
| $\Phi$ | volume-specific enthalpy |

## 1. INTRODUCTION

The common efficiency definition for positive displacement machines is given by ISO 4391:1984 [1]. However, the overall efficiency definition and the definitions of the partial efficiencies, namely the volumetric efficiency and mechanical-hydraulic efficiency are physically consistent only for an incompressible flow with the density $\varrho = \mathrm{const.}$ If the machine operates at high pressure levels the compressibility of the fluid and the dead volume of a pump must be taken into account. On this point, ISO 4391:1984 is physically inconsistent.

Achten et. al. [2] address this issue in their paper at FPMC 2019 presenting a critical review of ISO 4409:2007 [3]. ISO 4409:2007 specifies the procedure of efficiency measurements and adopts the efficiency definition from ISO 4391:1984. Meanwhile, a new version of ISO 4409 from 2019 exists (ISO 4409:2019 [4]), that no longer explicitly states efficiency definitions and instead only refers to ISO 4391:1984. Consequently, a critical review of efficiency definitions must address ISO 4391:1984. Achten et. al. introduce a new definition of the overall efficiency as well as the mechanical-hydraulic efficiency discussing the influence of the compressibility of a fluid as well as the dead volume of a positive displacement machine. At the same time, they question the validity of a volumetric efficiency definition.

Li and Barkei [5] continue on this issue in their paper at FPMC 2020 and also give new definitions of the overall and partial efficiencies considering a compressible flow. They introduce a new quantity $\Phi$ which designates the volume specific enthalpy of the conveyed fluid and serves as the equivalent to $\Delta p$ considering the efficiency definitions for an incompressible flow. Furthermore, their approach makes no assumptions regarding compressibility or the relation between pressure and density. Based on a comparison of their own efficiency definition with the definition of Achten et. al and Williamson and Manrig [6], Li and Barkei still find differences and inconsistencies among the results.

The motivation of this paper is to contribute to the ongoing discussion. In this regard, our argumentation adheres the following principles:

(i) Definitions are never wrong. Instead, appropriate criteria for definitions are their meaningfulness, their physical consistency and their acceptance. For efficiency definitions to be accepted, they must be easy to apply, practical for users, and based on a transparent derivation.

(ii) The definition of partial efficiencies based on the extension of the isentropic efficiency definition with the displacement volume goes hand in hand with the idea of an ideal, i.e. loss-free, machine that is characterized by its displacement volume. This allows the calculation of the converted energy per rotation as well as the volume flow of an ideal machine, which are essential for the partial efficiencies. Thus, the partial efficiencies have a high practical value and provide a starting point to modeling the overall efficiency. Modeling succeeds on the basis of loss analysis, as systematically started by Wilson [7] by means of tribology and fluid mechanics.

(iii) The idea of using an ideal machine as a reference is a proven and well-known approach. Four prominent examples demonstrate this: firstly, the considerations of Sadi Carnot on an ideal heat engine leading to the definition of Carnot's efficiency [8], secondly, the considerations of Betz on the upper limit of wind power for wind turbines [9], thirdly, the considerations of Pelz on the upper limit for hydropower in an open-channel flow [10] and, fourthly, the considerations of Turing on an abstract machine based on mathematical model, i.e. the Turing machine [11].

Following these principles, we present meaningful and physically consistent representations for the overall efficiency as well as the mechanical-hydraulic and the volumetric efficiencies.

At the beginning in section 2, we give with the most general efficiency definition which is the starting point of our considerations. In the following, we make assumptions concerning the fluid properties which are analogous to the ones made by Achten et. al., namely the linearization of the constitutive relation between pressure and density and the use of an averaged isentropic bulk modulus. In section 3, we introduce the effective displacement volume and analyze the energy conversion in an ideal positive displacement machine based on both extensive and intensive quantities of the converted energy. Consequently, we obtain physically consistent and meaningful representations of the overall efficiency, the volumetric efficiency and the mechanical-hydraulic efficiency. Section 4 compares the results of this paper with the definitions of Achten et. al. and Li and Barkei. Finally, section 5 gives the conclusion.

## 2. ENERGY BALANCE AND ASSUMPTIONS FOR EFFICIENCY DEFINITIONS

The assessment of the energy conversion in a positive displacement machine is based on the assumption that the machine operates stationary on a time-averaged basis. Hence, the first law of thermodynamics reads

$$P_S + \dot{Q} = \dot{m}\Delta h_{\mathrm{t}}, \tag{1}$$

with the mass flow $\dot{m}$, the difference of the mass-specific total enthalpy between machine outlet and inlet $\Delta h_{\mathrm{t}}$, the mechanical shaft power $P_{\mathrm{S}} = 2\pi M_S n$ being the product of the shaft torque $M_{\mathrm{S}}$ and the rotational speed $n$, and the heat flow $\dot{Q}$. All quantities are considered to be averaged over time.

In the case of a pump, $P_\mathrm{S}$ and $\Delta h_\mathrm{t}$ are both greater than zero, in the case of a motor, $P_\mathrm{S}$ and $\Delta h_\mathrm{t}$ are negative. The mass flow at the inlet and outlet of a machine are identical. In case of an external leakage $\dot{m}_\mathrm{L}$ it is assumed to be redirected to the inlet of the pump or the outlet of a motor respectively as shown in FIGURE 1. Due to environmental constraints, real external leakage is unlikely.

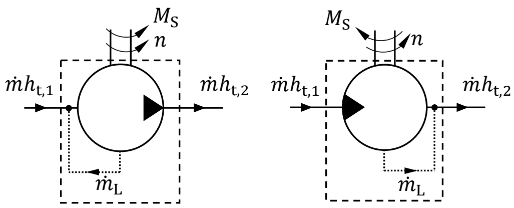



**FIGURE 1**: FIRST LAW OF THERMODYNAMICS FOR AN ADIABATIC POSITIVE DISPLACEMENT PUMP AND MOTOR.

The commonly used efficiency definition for positive displacement machines is the isentropic efficiency. In fact, this efficiency definition is used for all machines, turbo machines or positive displacement machines with a compressible flow or an incompressible flow, as long as the machine operates adiabatically. Considering an adiabatic machine, the isentropic efficiency $\eta$ is defined as the ratio of the product of mass flow and mass-specific isentropic and total enthalpy difference $\Delta h_{\mathrm{t,s}}$ and the shaft power

$$\eta := \left(\frac{\dot{m}\Delta h_{\mathrm{t,s}}}{P_\mathrm{S}}\right)^{\pm 1}. \tag{2}$$

The exponent $+1$ applies to pumps, the exponent $-1$ applies to motors. While the following sections focus on pumps, the described procedure can be applied on motors similarly.

Dividing the mass-specific total enthalpy difference $\Delta h_\mathrm{t}$ into the isentropic fraction $\Delta h_{\mathrm{t,s}}$ and the loss fraction $h_\mathrm{l}$, we obtain the following representation of the isentropic efficiency

$$\eta := 1 - \frac{\dot{m}h_\mathrm{l}}{P_\mathrm{S}}. \tag{3}$$

Equation (3) illustrates that the efficiency is a measure of the dissipative power losses $P_\mathrm{loss} = \dot{m}h_\mathrm{l}$.

Given an approximately incompressible flow and an ideally rigid machine, the total enthalpy $h_\mathrm{t}$ is

$$\begin{aligned} h_\mathrm{t} &= \frac{p}{\varrho} + \frac{u^2}{2} + gz + e, \\ \varrho h_\mathrm{t} &= p + \varrho\frac{u^2}{2} + \varrho gz + \varrho e \end{aligned} \tag{4}$$

equation (2) leads to the ISO 4391:1984 efficiency definition (the difference of kinetic energy $u^2/2$ and potential energy $gz$ are neglected).

$$\eta := \frac{\Delta pQ}{P_\mathrm{S}} = \frac{\Delta pQ}{2\pi M_\mathrm{S} n}, \tag{5}$$

with the pressure difference $\Delta p$ and the volume flow $Q$. Extending equation (5) with the displacement volume $V$, the efficiency can be written as the product of the volumetic efficiency $\eta_\mathrm{vol}$ and the mechanical-hydraulic efficiency $\eta_\mathrm{mh}$

$$\eta = \eta_\mathrm{vol}\eta_\mathrm{mh}, \qquad \eta_\mathrm{vol} := \frac{Q}{nV}, \qquad \eta_\mathrm{mh} := \frac{\Delta pV}{2\pi M_\mathrm{S}}. \tag{6}$$

The displacement volume needs to be determined experimentally on the basis of Toet's method [12].

In the case of high pressure differences, the mass-specific isentropic internal energy difference $\Delta e_\mathrm{s}$ must not be neglected which represents the converted energy due to compression. Hence, the compressibility of the fluid needs to be taken into account. At this point, we make the following two assumptions:

(i) The compression and decompression of the fluid is isentropic ($s = \mathrm{const}$) and can be described using an averaged isentropic bulk moduls $\bar{K}_\mathrm{S}$ or averaged isentropic compressibility $\bar{\kappa} = 1/\bar{K}_\mathrm{S}$.

(ii) The relation between volume $V$ or density $\varrho$ and pressure $p$ of a fluid

$$\bar{\kappa} := -\frac{1}{V}\frac{\mathrm{d}V}{\mathrm{d}p}\Big|_s = \frac{1}{\varrho}\frac{\mathrm{d}\varrho}{\mathrm{d}p}\Big|_s \tag{7}$$

is linearized and yields

$$\bar{\kappa} \approx -\frac{1}{V}\frac{\Delta V}{\Delta p} = \frac{1}{\varrho}\frac{\Delta\varrho}{\Delta p}. \tag{8}$$

As can be seen in section 3.2, these assumptions are not mandatory but can be easily extended by pressure dependent compressibility $\kappa(p)$. However, the assumptions shorten the efficiency representations derived from the isentropic efficiency definition in equation (2), as can be seen in the next section. Furthermore, Ivantysyn und Ivantysynova [13] state that the resulting error due to linearization for common hydraulic fluids is negligible.

## 3. REPRESENTATIONS OF THE EFFICIENCY DEFINITIONS

Based on the above assumptions, the next step is to determine the numerator of the isentropic efficiency definition in equation (2), which is the product of mass flow and the difference of the mass-specific isentropic enthalpy $\Delta h_{\mathrm{s}}$ (kinetic energy $u^2/2$ and potential energy $gz$ are neglected). In section 3.1 we follow the common approach to determine the numerator based on an analysis of an ideal positive displacement machine's cycle in the $p$-$V$-diagram. Section 3.2 gives the same result, however, considering the more meaningful representation of the ideal cycle with intensive; i.e. mass-specific, quantities in the $p$-$v$-diagram. In section 3.3 we extend the equation of the isentropic efficiency definition with the effective displacement volume which leads to definitions of the volumetric efficiency and the mechanical-hydraulic efficiency. This step is discussed in the context of our understanding of an ideal positive displacement machine.

### 3.1 Cycle of an ideal machine based on extensive quantities

Firstly and for reasons of clarity, we focus on the isentropic enthalpy difference $\Delta H_{\mathrm{s}}$ of the conveyed fluid mass per cycle, which is an extensive quantity. The isentropic enthalpy difference is equivalent to the energy transferred between an ideal machine and fluid per rotation. The time averaged mass flow

$$\dot{m} := \frac{1}{T}\int_0^T \tilde{\dot{m}}(t)\mathrm{d}t \tag{9}$$

is given by the time integral of the temporal mass flow $\tilde{\dot{m}}(t)$ and the cycle time $T = 1/n$. Hence, the time averaged mass flow is the product of conveyed fluid mass per rotation $m_{\mathrm{F}}$ and the rotational speed $n$

$$\dot{m} = n m_{\mathrm{F}}. \tag{10}$$

This yields

$$\dot{m}\Delta h_{\mathrm{s}} = n\Delta H_{\mathrm{s}}. \tag{11}$$

FIGURE 2 shows the cycle of an ideal positive displacement pump, i.e. loss-free and ideally rigid, with a dead volume $V_{\mathrm{d}}$ filled by a compressible fluid in a $p$-$V$-diagram. The shaded area, given by the points abcd, states the isentropic enthalpy difference $\Delta H_{\mathrm{s}}$ and needs to be calculated in order to derive a meaningful efficiency representation based on the efficiency definition (2) and equation (11). The dead volume results from the design of a positive displacement pump and must be calculated on the basis of the geometric pump dimensions. The displacement volume $V$ is determined experimentally at a pressure difference $\Delta p = 0$ (cf. [12]).

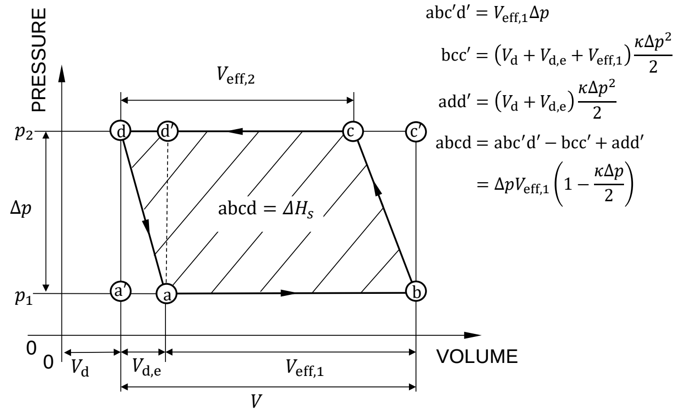



**FIGURE 2**: IDEAL CYCLE FOR A POSITIVE DISPLACEMENT PUMP WITH A DEAD VOLUME AND A COMPRESSIBLE FLUID.

Beginning the ideal cycle at the top dead center, point d, and the pressure $p_2$, the dead volume decompresses, d→a, before refilling the displacement chamber at the pressure level $p_1$, d→a. The difference between the compressed and expanded dead volume is called $V_{\mathrm{d,e}}$. In the following, the fluid with the effective displacement volume $V_{\mathrm{eff,1}}$ flows into the displacement chamber, a→b. At the bottom dead center, point b, the total volume $V_{\mathrm{t}}$ of the displacement chamber is

$$V_{\mathrm{t}} = V_{\mathrm{d}} + V = V_{\mathrm{d}} + V_{\mathrm{d,e}} + V_{\mathrm{eff,1}}. \tag{12}$$

The effective displacement volume $V_{\mathrm{eff,1}}$ is given by

$$V_{\mathrm{eff,1}} = V - V_{\mathrm{d,e}} \tag{13}$$

and equation (8) yields

$$V_{\mathrm{d,e}} = V_{\mathrm{d}}\bar{\kappa}|\Delta p|\,. \tag{14}$$

Consequently, the effective displacement volume $V_{\mathrm{eff,1}}$ can be calculated from the experimentally determined displacement volume $V$, the geometrically calculated dead volume $V_{\mathrm{d}}$, the averaged isentropic compressibility $\bar{\kappa}$ and the pressure difference $\Delta p$. This is of major importance, as the decompression of the dead volume and reduction of usable displacement volume $\Delta V = V_{\mathrm{d,e}} = V - V_{\mathrm{eff,1}}$ does not cause volumetric losses or dissipation of energy. Instead, it underlines the effective Volume $V_{\mathrm{eff,1}}$ being the relevant geometric quantity in the partial efficiencies, the volumetric and mechanical-hydraulic efficiency.

Further on, the total volume is compressed to the pressure level $p_2$, b→c, and displaced from of the displacement chamber until the top dead center is reached again, c→d.

The isentropic enthalpy difference $\Delta H_s$ (abcd) can now be calculated from the following areas, each described by its corner points

$$\mathrm{abcd} = \mathrm{abc'd'} - \mathrm{bcc'} + \mathrm{add'}. \tag{15}$$

Each area can be easily calculated based on the edge lengths. These correspond to the pressure difference $\Delta p$, the effective displacement volume $V_{\mathrm{eff},1}$ and the volume difference due to compression, b→c, or expansion, d→a, calculated with equation (8). At this point, it must be emphasized that the compression, b→c and expansion, d→a can only be calculated under the assumption of a closed control volume and, thus, a constant mass. The results for the different areas are

$$\begin{aligned} \mathrm{abc'd'} &= \Delta \mathrm{p} V_{\mathrm{eff},1}\,, \\ \mathrm{bcc'} &= \left(\mathrm{V_d} + \mathrm{V_{d,e}} + V_{\mathrm{eff},1}\right)\frac{\bar{\kappa}\Delta p^2}{2}\,, \\ \mathrm{add'} &= \left(\mathrm{V_d} + \mathrm{V_{d,e}}\right)\frac{\bar{\kappa}\Delta p^2}{2}. \end{aligned} \tag{16}$$

The area $\mathrm{add'} = \mathrm{ada'}$ is calculated from the perspective of compressing the decompressed dead volume $V_{\mathrm{d}} + V_{\mathrm{d,e}}$, a→d. In this way, the compression energy can be represented by $\mathrm{V_d} + \mathrm{V_{d,e}}$. Since the compression or decompression is assumed to be isentropic, the absolute value of the converted mechanical energy is equal, a→d = d→a. However, due to the assumption made, namely the linearized relation in equation (8), there is a deviation between expansion and compression:

$$\text{area add'}\ \text{expansion (d→a)} = \frac{V_{\mathrm{d}}\bar{\kappa}\Delta p^2}{2}, \tag{17}$$

$$\text{area add'}\ \text{compression (a→d)} = \frac{\left(V_{\mathrm{d}} + V_{\mathrm{d,e}}\right)\bar{\kappa}\Delta p^2}{2}. \tag{18}$$

This deviation results from the linearization error which is negligible for the range of practical pressures and therefore not considered any further. Equations (15) and (16) now leads to the isentropic enthalpy difference $\Delta H_{\mathrm{S}}$ of the conveyed fluid mass $m_{\mathrm{eff}} = V_{\mathrm{eff},1}\varrho_1$ per rotation

$$\mathrm{abcd} = \Delta p V_{\mathrm{eff},1}\left(1 - \frac{\bar{\kappa}\Delta p}{2}\right). \tag{19}$$

Equation (19) gives a short, meaningful and physically consistent representation of the isentropic enthalpy difference which can be used for the efficiency representation based on the definition in equation (2). Before that, we derive the same result based on the mass specific isentropic enthalpy, which is an intensive quantity.

### 3.2 Cycle of an ideal machine based on intensive quantities

The ideal cycle on the basis of the mass-specific volume presented in FIGURE 3 leads to the mass-specific enthalpy. The mass-specific representation is advantageous since it allows the determination of the mass-specific isentropic enthalpy difference which can be used directly for the definition of the efficiency (cf. equation (2)). The mass-specific isentropic enthalpy difference from state 1 to 2 yields

$$\Delta h_s = \int_1^2 v\mathrm{d}p \approx \frac{\Delta p}{\varrho_1}\left(1 - \frac{\bar{\kappa}\Delta p}{2}\right). \tag{20}$$

As can be seen, the introduction of the displacement volume is not necessary.

Equation (19) can also be derived from the mass-specific $p$-$v$-diagram shown in FIGURE 3. One obtains the isentropic enthalpy difference $\Delta H_{\mathrm{s}}$ (abcd) by multiplying the corresponding fluid masses of the dead volume $m_{\mathrm{d}}$ and the effectively conveyed volume $m_{\mathrm{eff}} = \varrho_1 V_{\mathrm{eff},1}$

$$\begin{aligned} \mathrm{abcd} = \Delta H_{\mathrm{s}} &= (m_{\mathrm{eff}} + m_{\mathrm{d}})\Delta h_{\mathrm{s}} - m_{\mathrm{d}}\Delta h_{\mathrm{s}} \\ &= m_{\mathrm{eff}}\Delta h_{\mathrm{s}} \\ &= \Delta p V_{\mathrm{eff},1}\left(1 - \frac{\bar{\kappa}\Delta p}{2}\right). \end{aligned} \tag{21}$$

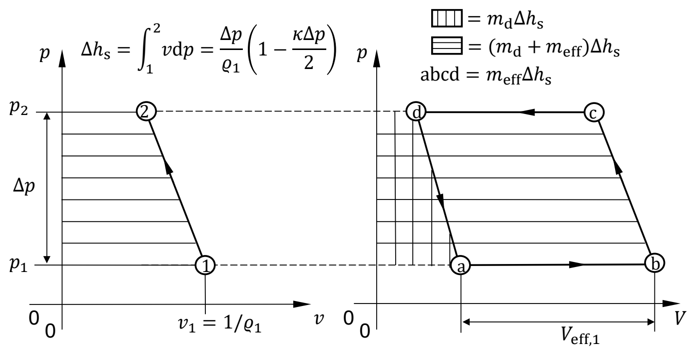



**FIGURE 3:** MASS-SPECIFIC $p$-$v$-DIAGRAM FOR AN IDEAL POSITIVE DISPLACEMENT PUMP AND A COMPRESSIBLE FLUID.

At the same time, it is obvious that the mass-specific isentropic enthalpy difference based on equation (20) can also be calculated with a non-linearized relationship of pressure and density (cf. equation (7)) and a pressure dependent compressibility $\kappa(p)$.

### 3.3 Efficiency representations

Following the efficiency definition according to definition (2) and equation (20) one obtains

$$\eta \coloneqq \frac{\dot{m}\Delta h_s}{2\pi M_S n} = \frac{Q_1 \Delta p}{2\pi M_S n}\left(1 - \frac{\bar{\kappa}\Delta p}{2}\right). \tag{22}$$

The mass flow $\dot{m} = \varrho_1 Q_1$ is the product of the volume flow $Q_1$ and the density $\varrho_1$ at the pump inlet. Since the volume flow is usually measured at the pump outlet, the volume flow $Q_1$ can be calculated with equation (8) by

$$Q_1 = \frac{Q_2}{1 - \bar{\kappa}\Delta p}. \tag{23}$$

The derivation of representations of the partial efficiencies succeeds based on equation (22) and the introduction of the effective displacement volume $V_{\mathrm{eff},1}$. Extending equation (22) with the effective displacement volume $V_{\mathrm{eff},1}$ (cf. equation (13) and (14)) in the numerator and denominator, the isentropic efficiency yields

$$\eta \coloneqq \frac{Q_1}{nV_{\mathrm{eff},1}} \frac{\Delta p V_{\mathrm{eff},1}}{2\pi M_S}\left(1 - \frac{\bar{\kappa}\Delta p}{2}\right). \tag{24}$$

Consequently, definitions of the volumetric efficiency $\eta_{\mathrm{vol}}$ and the mechanical-hydraulic efficiency $\eta_{\mathrm{mh}}$ can be given by

$$\begin{aligned} \eta_{\mathrm{vol}} &\coloneqq \frac{Q_1}{nV_{\mathrm{eff},1}} = \frac{Q_1 \Delta p \left(1 - \frac{\bar{\kappa}\Delta p}{2}\right)}{nV_{\mathrm{eff},1}\Delta p \left(1 - \frac{\bar{\kappa}\Delta p}{2}\right)}, \\ \eta_{\mathrm{mh}} &\coloneqq \frac{\Delta p V_{\mathrm{eff},1}}{2\pi M_S}\left(1 - \frac{\bar{\kappa}\Delta p}{2}\right) = \frac{\Delta H_S}{2\pi M_S} \end{aligned} \tag{25}$$

At this point, one must understand that this approach goes hand in hand with the idea of an ideal machine. The ideal machine is characterized by the effective displacement volume $V_{\mathrm{eff},1}$, the ideal volume flow $Q_{\mathrm{eff},1} = nV_{\mathrm{eff},1}$ and the loss-free energy transferred between machine und fluid, i.e. the isentropic enthalpy $\Delta H_S = \Delta p V_{\mathrm{eff},1}\left(1 - \frac{\bar{\kappa}\Delta p}{2}\right)$.

Thereby, the volumetric efficiency represents the ratio of the volume flow at the inlet $Q_1$ to the ideal volume flow $Q_{\mathrm{eff},1}$. This is equivalent to the ratio of the hydraulic power of the conveyed fluid by the real machine to the hydraulic power of the conveyed fluid by the ideal machine. The mechanical-hydraulic efficiency represents the ratio of the loss-free energy transferred between ideal machine and fluid to the shaft work of the real machine during one rotation.

Furthermore, this approach allows to calculate volumetric and mechanical-hydraulic losses. These losses are the difference between ideal and real machines behaviour and can be given in a physically consistent and meaningful way as follows:

(i) The leakage $Q_{\mathrm{L}} = Q_{\mathrm{eff},1} - Q_1$ represents the difference between the effective or ideal volume flow $Q_{\mathrm{eff},1} = nV_{\mathrm{eff},1}$ and the measured volume flow at pump inlet $Q_1$. Hence, the leakage causes the power loss $P_{\mathrm{loss,L}} = \Delta p Q_{\mathrm{L}}\left(1 - \frac{\bar{\kappa}\Delta p}{2}\right)$. FIGURE 4 shows the energy loss due to leakage as a marked area ($b'bcc'$) in a $p$-$V$-diagramm. This is the energy transferred to the fluid volume $V_{\mathrm{L}} = V_{\mathrm{eff},1} - Q_1/n$ and which is lost due to leakage. This representation is based on the idea that, in the case of a pump, leakage occurs after the energy is transferred from machine to the fluid. It does not matter if the real machine's leakage behavior is different as the leakage is a calculated quantity based on the ideal volume flow. Furthermore, it is consistent with our approach assuming a closed control volume for the compression and expansion presented in FIGURE 2.

(ii) The friction torque $M_{\mathrm{mh}} = M_S - M_{\mathrm{hyd}}$ resulting from friction and momentum losses of the conveyed fluid is calculated from the difference of shaft torque $M_S$ and hydraulic torque $M_{\mathrm{hyd}} = \frac{\Delta H_S}{2\pi} = \frac{\Delta p V_{\mathrm{eff},1}}{2\pi}\left(1 - \frac{\bar{\kappa}\Delta p}{2}\right)$. This results in the power loss $P_{\mathrm{loss,mh}} = 2\pi M_{\mathrm{mh}} n$. Similar to the leakage, the friction torque is a calculated quantity based on the loss-free energy, i.e. the isentropic enthalpy $\Delta H_S$, transferred between ideal machine and fluid.

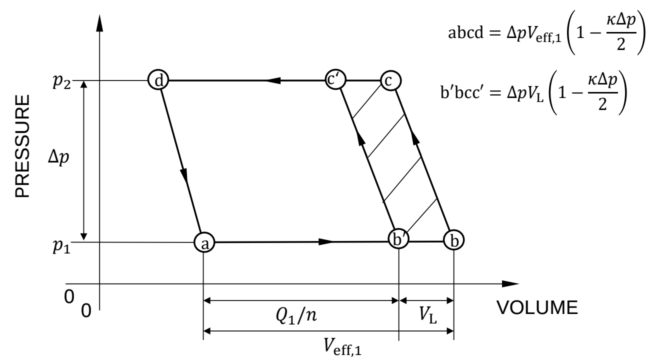



**FIGURE 4**: ENERGY LOSS DUE TO LEAKAGE REPRESENTED IN A $p$-$V$-DIAGRAM.

Equations (22) and (25) provide representations of the overall efficiency, the volumetric and the mechanical-hydraulic efficiency which measure the energetic quality of positive displacement pumps with a dead volume in a physically consistent and meaningful way. At the same time, these partial efficiencies can also be quantified based on the volumetric losses $Q_{\mathrm{L}}$ and the friction torque $M_{\mathrm{mh}}$

$$\eta_{\mathrm{vol}} \coloneqq 1 - \frac{Q_{\mathrm{L}}}{nV_{\mathrm{eff},1}}, \tag{26}$$

$$\eta_{\mathrm{mh}} := \frac{1}{1 + \frac{2\pi}{1 - \bar{\kappa}\,\Delta p/2}\frac{M_{\mathrm{mh}}}{\Delta p V_{\mathrm{e}}}}$$

If the dead volume is negligibly small or the flow is approximately incompressible due to low pressure differences of the pump, the effective displacement volume $V_{\mathrm{eff,1}}$ and the experimentally determined displacement volume $V$ will be identical. Furthermore, $\kappa\Delta p \ll 1$ and the efficiency definitions of ISO 4391 according to equations (5) and (6) can be applied.

## 4. COMPARISON OF EFFICIENCY REPRESENTATIONS IN THE LITERATURE

As stated in the introduction the motivation of this paper is to contribute to the ongoing and fruitful discussion about meaningful and physically consistent efficiency representations of positive displacement machines with a dead volume and a compressible flow which was started at FPMC 2019 by Achten et. al [2].

Against this background, the overall efficiency and partial efficiency representations derived in this paper are compared to the efficiency representations given by Achten et. al. and Li and Barkei [4]. TABLE 1 summarizes all efficiency representations in the notation of this paper considering a pump with one single displacement chamber.

Achten et. al. make new proposals for the overall and the mechanical hydraulic efficiency. They also calculate the isentropic enthalpy $\Delta H_{\mathrm{S}}$ from the ideal cycle of a positive displacement machine (cf. FIGURE 2) but derive a sightly different formula resulting in a different representation of the mechanical efficiency. This is due to the calculation of the area add′ (cf. FIGURE 2) from the perspective of an expansion (see equations (17) and (18)) and due to the linearization error. Hence, the differences for mechanical hydraulic efficiency representation are negligible. On the other hand, Achten et. al.'s definition of the overall efficiency is inconsistent with the definition of the mechanical hydraulic efficiency. They integrate the inner energy (see [2] equation (5)) under the assumption of a mean density $\bar{\varrho}$ which is approximately $\bar{\varrho} = \varrho_1 = \varrho_2$ (see [2] equation (6)). This assumption is neither transparently presented nor consistent with their assumption of a pressure dependent density regarding the cycle of an ideal positive displacement machine (see [2] FIGRUE 2)). In fact, Achten et. al. consider the overall efficiency and the mechanical-hydraulic efficiency independently of each other in contrast to this paper. At the same time, they question the validity of a volumetric efficiency definition and, thus, do not provide one. A physically consistent volumetric efficiency definition that fulfills $\eta = \eta_{\mathrm{mh}}\eta_{\mathrm{vol}}$ and that is based on the idea of an ideal and reference machine is not achievable due to their inconsistent integration of the inner energy. In summary, they apply the idea of an ideal machine in the context of their mechanical-hydraulic efficiency definition, but and in contrast to this paper not to the volumetric efficiency definition and volumetric losses.

Li and Barkei give generally valid and consistent definitions of the overall efficiency, the volumetric and the mechanical hydraulic efficiency. These definitions contain their newly introduced quantity $\Phi$ which is the volume-specific enthalpy

$$\Phi := \frac{\Delta H_{\mathrm{S}}}{V_{\mathrm{eff,2}}}. \tag{27}$$

In this way, they do not make any assumptions regarding the fluid properties, namely the compressibility of the fluid, e.g. by using an averaged bulk modulus or by linearizing the relation of pressure and density. However, this is why their approach results in efficiency representations that are slightly more difficult to understand. Regardless of this, Li and Barkei's representations are identical to the representations derived in this paper when taking into account the assumptions made in section 2.

## 5. CONCLUSION

On the basis of the most general efficiency definition, namely the isentropic efficiency, transparent assumptions considering the fluid properties and the $p$-$v$ diagram as well as the idea of an ideal and reference positive displacement machine, we derive physically consistent and meaningful representations of the overall, the volumetric and the mechanical-hydraulic efficiency. At the same time, these representations fulfill the requirements for a high acceptability, as they are easy to apply, practical for users, and based on a transparent deviation. These representations are consistent with the definitions of Li and Barkei [4] and may serve as a template for a revision of ISO 4391:1984 [1]. In particular, the use of the effective volume $V_{\mathrm{eff,1}}$ (cf. equation (13) and (14)) is the basis of a short and meaningful efficiency representation.

**TABLE 1**:EFFICIENCY DEFINITIONS OF PUMPS.

| volumetric efficiency | |
|---|---|
| Achten et. al: | no definition |
| Li and Barkei: | $\eta_{\mathrm{vol}} := \frac{Q_1}{nV_{\mathrm{eff},1}} = \frac{Q_2}{nV_{\mathrm{eff},2}}$ |
| this paper: | $\eta_{\mathrm{vol}} := \frac{Q_1}{nV_{\mathrm{eff},1}} = \frac{Q_2}{nV_{\mathrm{eff},2}}$ |
| **mechanical-hydraulic efficiency** | |
| Achten et. al: | $\eta_{\mathrm{mh}} := \frac{\Delta pV}{2\pi M_{\mathrm{S}}}\left[1-\Delta p\bar{\kappa}\left(\frac{1}{2}+\frac{V_{\mathrm{d}}}{V}\right)\right]$ |
| Li and Barkei: | $\eta_{\mathrm{mh}} := \frac{V_{\mathrm{eff},2}\Phi(\Delta p, p_1)}{2\pi M_{\mathrm{S}}}$ <br> $\Phi(\Delta p, p_1) \coloneqq \frac{1}{V_{\mathrm{eff},2}}\int_1^2 V\mathrm{d}p = \frac{\Delta H_{\mathrm{S}}}{V_{\mathrm{eff},2}}$ |
| this paper: | $\eta_{\mathrm{mh}} \coloneqq \frac{\Delta pV_{\mathrm{eff},1}}{2\pi M_{\mathrm{S}}}\left(1-\frac{\bar{\kappa}\Delta p}{2}\right) = \frac{\Delta H_{\mathrm{S}}}{2\pi M_{\mathrm{S}}}$ <br> $= \frac{\Delta pV_{\mathrm{eff},2}}{2\pi M_{\mathrm{S}}}\frac{(1-\bar{\kappa}\Delta p/2)}{(1-\bar{\kappa}\Delta p)}$ |
| **overall efficiency** | |
| Achten et. al: | $\eta := \frac{\mathrm{p}_2 Q_2\left(1+\frac{p_2\bar{\kappa}}{2}\right)-p_1 Q_1}{2\pi M_{\mathrm{S}} n}$ |
| Li and Barkei: | $\eta := \frac{Q_2\Phi(\Delta p, p_1)}{2\pi M_{\mathrm{S}} n}$ |
| this paper: | $\eta \coloneqq \frac{\Delta pQ_1}{2\pi M_{\mathrm{S}} n}\left(1-\frac{\bar{\kappa}\Delta p}{2}\right)$ <br> $= \frac{\Delta pQ_2}{2\pi M_{\mathrm{S}} n}\frac{(1-\bar{\kappa}\Delta p/2)}{(1-\bar{\kappa}\Delta p)}$ |

## REFERENCES


[1] ISO 4391:1984, “Hydraulic fluid power; pumps, motors and integral transmissions, parameter definitions and letter symbols”, Beuth Verlag, Berlin, Germany, 1984.

[2] P. Achten, R. Mommers, T. Nishiumi, H. Murrenhoff, N. Sepehri, K. Stelson, J.-O. Palmberg, and K. Schmitz, “Measuring the losses of hydrostatic pumps and motors: A critical review of Iso4409:2007,” in Proceedings of 2019 ASME/BATH Symposium on Fluid Power and Motion Control. American Society of Mechanical Engineers, 2019.

[3] ISO 4409:2007, “Hydraulic fluid power – Positive displacement pumps, motors and integral transmissions – Methods of testing and presenting basic steady state performance”, Beuth Verlag, Berlin, Germany, 2007.

[4] ISO 4409:2019, “Hydraulic fluid power – Positive displacement pumps, motors and integral transmissions – Methods of testing and presenting basic steady state performance”, Beuth Verlag, Berlin, Germany, 2007.

[5] P.Y. Li and J.H. Barkei, “Hydraulic effort and the efficiencies of pump and motors with compressible fluid” in Proceedings of 2020 ASME/BATH Symposium on Fluid Power and Motion Control. American Society of Mechanical Engineers, 2020.

[6] C. Williamson and N. Manring, “A more accurate definition of mechanical and volumetric efficiencies for digital displacement pumps” in Proceedings of 2019 ASME/BATH Symposium on Fluid Power and Motion Control. American Society of Mechanical Engineers, 2019.

[7] W. E. Wilson, “Positive-displacement Pumps and Fluid Motors”, New York, Pitman Publishing Corporation, 1950.

[8] N.L. Sadi Carnot, „Réflexions sur la puissance mortice du feu et sur les machines propres à développer cette puissance”, Bachelier, Paris, 1824 (in French)

[9] A. Betz, “Das Maximum des theoretisch möglichen Ausnutzung des Windes durch Windmotoren“, in Zeitschrift für das gesamte Turbinenwesen, 1920 (in German)

[10] P.F. Pelz, „Upper Limit for Hydropower in an Open-Channel Flow”, in Journal of Hydraulic Engineering, 2011

[11] A. M. Turing, “On Computable Numbers, with an Application to the Entscheidungsproblem”, in Proceedings of the London Mathematical Society 2.1, 1937

[12] G. Toet, J. Johnson, J. Montague, K. Torres and J. Garcia-Bravo, „The Determination of the Theoretical Stroke Volume of Hydrostatic Positive Displacement Pumps and Motors from Volumetric Measurements“, in *Energies 12.3*, 2019.

[13] J. Ivantysyn und M. Ivantysynova, "Hydrostatische Pumpen und Motoren: Konstruktion und Berechnung“, Vogel, Würzburg, Germany, 1993.

4. First rebuttal of Christian Schänzle and Peter Pelz to the first review of Peter Achten and Robin Mommers Note_210805_Response_Review_AchtenEtAl (first rebuttal, received on August 5, 2021)

Dear Peter Achten,

Dear Robin Mommers,

thank you very much for your structured and detailed review. We can see that a lot of effort was put into your review and that the topic is of major importance to you, as it is to us, too. We find your non-anonymized review courageous and in the spirit of an open and constructive discussion. We appreciate that very much.

Nevertheless, different views come to light, which are inherent in science and have to be endured by the opposing position. This applies all the more to the topic of efficiency, since the efficiency is a <u>defined</u> quantity.

At this point, it is important for us to name our three guiding principles that guide us in our argumentation:

i. Definitions are never wrong. Instead, appropriate criteria for definitions are their meaningfulness, their physical consistency and their acceptance. For efficiency definitions to be accepted, they must be easy to apply, practical for users, and based on a transparent derivation.
ii. The definition of partial efficiencies based on the extension of the isentropic efficiency definition with the displacement volume goes hand in hand with the idea of an ideal, i.e. loss-free, machine that is characterized by its displacement volume. This allows the calculation of the converted energy per rotation as well as the volume flow of an ideal machine, which are essential for the partial efficiencies. Thus, the partial efficiencies have a high practical value and provide a starting point to modeling the overall efficiency. Modeling succeeds on the basis of loss analysis, as systematically started by Wilson by means of tribology and fluid mechanics.
iii. The idea of using an ideal machine as a reference is a proven and well-known approach. Four prominent examples demonstrate this: firstly, the considerations of Sadi Carnot on an ideal heat engine leading to the definition of Carnot's efficiency, secondly, the considerations of Betz on the upper limit of wind power for wind turbines, thirdly, the considerations of Pelz on the upper limit for hydropower in an open-channel flow and, fourthly, the considerations of Turing on an abstract machine based on mathematical model, i.e. the Turing machine.

The acceptance of definitions is decided by a research and industry community, in the case of an efficiency definition finally by an ISO committee. There may be different views and opinions on the meaningfulness of definitions, e.g. the definition of volumetric efficiency, which may not be dispelled. However, there should be an agreement on the physical consistency, as this is based on axioms such as the first law, material laws such as a compressible fluid with an isentropic change of state and model assumptions or simplifications such as linearization. This provides a transparent argumentation on the basis of which the acceptance of a definition can be decided. This is the spirit in which our paper was written and, in this spirit, we are pleased to respond to your review. Moreover, you find our revised paper including yellow highlighting of the revised passages.

Christian Schänzle and Peter Pelz

**Response to your introduction and consensus**

We agree with your motivation, your goals and with your view on model assumptions, as also transparently set out in our paper. Furthermore, we agree that the differences between your and our mechanical-hydraulic efficiency definition is due to the linearization error and, thus, the differences for mechanical hydraulic efficiency representation are negligible.

However, we are very critical of one of your basic assumptions that we do not want to follow. In our opinion, this assumption reveals your inconsistent argumentation:

Your derivation of the overall efficiency definition is inconsistent with the definition of the mechanical hydraulic efficiency. You integrate the inner energy (see your paper equation (5)) under the assumption of a mean density $\bar{\varrho}$ which is approximately $\bar{\varrho} = \varrho_1 = \varrho_2$ (see equation (6)). This assumption is neither transparently presented nor consistent with your assumption of a pressure dependent density (or volume) regarding the cycle of an ideal positive displacement machine (see your paper figure 2)). If your argument of a negligible error is made for equation (7) we do not understand why this should not also apply to figure 2 and equation (13) in your paper?

Whereas we define the partial efficiencies by extending the overall efficiency definition with the effective displacement volume, you consider the overall efficiency and the mechanical-hydraulic efficiency independently of each other. A consistent volumetric efficiency definition that fulfills $\eta = \eta_{\mathrm{mh}}\eta_{\mathrm{vol}}$ and that is based on the idea of an ideal and reference machine is not achievable due to your inconsistent integration of the inner energy.

Our commonly used approach extending the overall efficiency definition with the displacement volume goes hand in hand with the idea of an ideal machine. The ideal machine is characterized by the effective displacement volume $V_{\mathrm{eff,1}}$, the ideal volume flow $Q_{\mathrm{eff,1}} = nV_{\mathrm{eff,1}}$ and the loss-free energy transferred between machine und fluid, i.e. the isentropic enthalpy $\Delta H_{\mathrm{S}} = \Delta p V_{\mathrm{eff,1}}\left(1 - \frac{\bar{\kappa}\Delta p}{2}\right)$. Thereby, the volumetric efficiency represents the ratio of the volume flow at the inlet $Q_1$ to the ideal volume flow $Q_{\mathrm{eff,1}}$. This is equivalent to the ratio of the hydraulic power of the conveyed fluid by the real machine to the hydraulic power of the conveyed fluid by the ideal machine. The mechanical-hydraulic efficiency represents the ratio of the loss-free energy transferred between ideal machine and fluid to the shaft work of the real machine during one rotation.

This commonly used approach allows to calculate volumetric and mechanical-hydraulic losses. These losses are the difference between ideal and real machines behaviour and can be given in a physically consistent and meaningful way as follows:

(i) The leakage $Q_{\mathrm{L}} = Q_{\mathrm{eff,1}} - Q_1$ represents the difference between the effective or ideal volume flow $Q_{\mathrm{eff,1}} = nV_{\mathrm{eff,1}}$ and the measured volume flow at pump inlet $Q_1$. Hence, the leakage causes the power loss $P_{\mathrm{loss,L}} = \Delta p Q_{\mathrm{L}}\left(1 - \frac{\bar{\kappa}\Delta p}{2}\right)$. Figure 4 in our revised paper shows the energy loss due to leakage as a marked area $(b'bcc')$ in a $p$-$V$-diagramm. This is the energy transferred to the fluid volume $V_{\mathrm{L}} = V_{\mathrm{eff,1}} - Q_1/n$ and which is lost due to leakage. This representation is based on the idea that, in the case of a pump, leakage occurs after the energy is transferred from machine to the fluid. It does not matter if the real machine's leakage behavior is different as the leakage is a calculated quantity based on the ideal volume flow. Furthermore, it is consistent with the approach assuming a closed control volume for the compression and expansion considering the ideal cycle (see figure 2 in your and our paper).

(ii) The friction torque $M_{\mathrm{mh}} = M_{\mathrm{S}} - M_{\mathrm{hyd}}$ resulting from friction and momentum losses of the conveyed fluid is calculated from the difference of shaft torque $M_{\mathrm{S}}$ and hydraulic torque $M_{\mathrm{hyd}} = \frac{\Delta H_{\mathrm{S}}}{2\pi} = \frac{\Delta p V_{\mathrm{eff,1}}}{2\pi}\left(1 - \frac{\bar{\kappa}\Delta p}{2}\right)$. This results in the power loss $P_{\mathrm{loss,mh}} = 2\pi M_{\mathrm{mh}} n$. Similar to the leakage, the friction torque is a calculated quantity based on the loss-free energy, i.e. the isentropic enthalpy $\Delta H_{\mathrm{S}}$, transferred between ideal machine and fluid.

In our understanding, you apply the idea of an ideal machine in the context of your mechanical-hydraulic efficiency definition, but and in contrast the common approach not to the volumetric efficiency definition and volumetric losses.

**<u>Response to Differences</u>**

To 1.: Our consideration is valid for all pumps and motors, also with external drainage, as can be seen in our figure 1 and our chosen system boundary.

To 2.: Your argumentation is from a standpoint of an application. The efficiency definition is from a standpoint of a standardized procedure under standard conditions. A higher pressure at the inlet is possible, the question is whether this must be considered for the standardized efficiency measurements. Nevertheless, we see no limitation for our consideration.

To 3.: Please see "Response to introduction and consensus". Our understanding of the losses is different to your understanding. Losses can only be calculated when we have an idea of an ideal and reference machine. The losses are the difference between real machine behaviour and ideal machine behavior. Your standpoint only focuses on real machine behaviour and on details which may be correct but neglect the fact that the losses depend on the ideal machine bahaviour as well.

To 4.: see "Response to introduction and consensus".

To 5.: see "Response to comments 15-17" and "Response to introduction and consensus".

To 6: Please consider equations (20) and (21) as well as figure 3 in our revised paper. In our opinion, the mass-specific representation of the ideal cycle is more meaningful and a discussion of linearization errors become superfluent. One can argument based on the fluid masses of the dead volume and the conveyed fluid.

**<u>Response to Comments</u>**

To 1.: $V_{\mathrm{eff}}$ can be determined as stated in our revised paper by equations (13) and (14).

To 2.: No. We prefer our representations of the partial efficiency definitions based on $V_{\mathrm{eff}}$ because in our opinion they are shorter and more meaningful. This is illustrated by equations (20) and (21) as well as Figure 3.

To 3.: Your suggested equation is similar to ours. In our consideration $\dot{m}_1$ und $\dot{m}_2$ are equal due to our chosen system boundary (see Figure 1). We do not see a benefit in your suggested equation. In addition, the challenge becomes apparent when $\dot{m}_1$ und $\dot{m}_2$ are not equal. A calculation of the mass-specific internal energy difference $\Delta u$ requires the assumption of a constant mass and a closed control volume. You solve this challenge introducing a mean density which is inconsistent with your consideration of the ideal cycle.

To 4.: Thank you for this comment, we corrected this notation.

To 5.: It is possible, but we aim at a representation of $\Delta H_s$ based on $V_{\text{eff}}$. This is shown in figure 2 as well as figure 3. In our opinion, in particular figure 3 makes it easy to understand why $V_{\text{eff}}$ is the more meaningful quantity.

To 6.: No, we do not agree. Your introduction of a mean density is not transparent and you do not give an equation to calculate the mean density. Furthermore, it is not consistent with the consideration of the mechanical hydraulic efficiency. Even if the assumption of a mean density only leads to slight deviations, it results in inconsistent representations of the overall and mechanical hydraulic efficiency.

To 7.: Yes.

To 8. We prefer our representation of abcd.

To 9.: A definition is never wrong or impossible. We present our understanding of the volumetric efficiency and volumetric losses in detail in our revised paper and above (see "Response to introduction and consensus"). The definition of volumetric losses and of a volumetric efficiency in the context of the idea of an ideal and reference positive displacement machine is consistent with our understanding of the mechanical hydraulic efficiency.

To 10.: see 9

To 11.: see "Response to introduction and consensus" and 3.

To 12.: see "Response to introduction and consensus"

To 13.: see "Response to introduction and consensus". Furthermore, we do not assume a mean density (see equation (7) und (8) in our paper)

To 14.: see "Response to introduction and consensus".

To 15 - 17.: We do not mention the addressed statements anymore. They had been referred to your statement questioning the validity of the volumetric efficiency. For the rotary positive displacement pump manufacturers we cooperate with, the volumetric efficiency is more important than the mechanical hydraulic efficiency especially at a low fluid viscosity. Furthermore, the customer is usually interested in the volume flow for his application. This information is given by the volumetric efficiency.

To 18. See 1. and 2.

To 19.: see 9.

To 20. Yes. We make this statement in our revised paper.

To 21.: We do not see a mistake in our approach.

To 22.: Please consider our argumentation. Firstly, we specify the system boundary and apply the first law of thermodynamics, secondly, we define the overall efficiency that is consistent with the isentropic or adiabatic efficiency (commonly applied to all kinds of fluid energy machines), thirdly, we make assumptions concerning the fluid, fourthly, we extend the overall efficiency definition with the effective displacement volume. This fourth step goes hand in hand with the idea of an ideal and reference machine as discussed above (see "Response to introduction and consensus"). Your approach defining the mechanical hydraulic efficiency is also based on this idea. Whereas you follow this idea only for the definition of the mechanical hydraulic efficiency, we apply this idea also to the volumetric efficiency definition. Moreover, our view on the partial efficiencies results from the overall efficiency definition whereas you consider the overall and the partial efficiency independently of each other.

5. Comment on mass density (send by Robin Mommers on August 6, 2021 to the authors)

The following contains an explanation of the derivations that are shown in “Achten et al. (2019) Measuring the losses of hydrostatic pumps and motors: A critical review of Iso4409:2007”. Equation numbers are references to the equations in this paper. This explanation concerns eq.(1)-(7), and focuses on the mass density of the hydraulic oil.

## Presented method

From eq.(1)-(3), it follows that we are looking for a definition of $u_i$, which describes the specific internal energy of oil in state $i$ (with $i$ = 1 or $i$ = 2 in this case). This definition is found to be the differential function shown in eq.(4) which is integrated in eq.(5).

Since we need a second state to calculate this integral, we choose a state at which the energy is known to be very low (state 0 with $p_0$ = 0 bar). For simplicity, we indeed assume that the mass density in this integral is constant, but not to a mean density, but the mass density in state $i$. So perhaps, a clearer way to write eq.(5) is the following form:

$$u_i - u_0 = \frac{1}{\rho_i \bar{K}_s} \int_{p_0}^{p_i} p \, \mathrm{d}p \quad \rightarrow \quad u_i = \frac{p_i^2}{2\rho_i \bar{K}_s} \tag{a}$$

In other words, we use a <u>different</u> mass density for oil from state 1 and state 2. As you mentioned, this indeed is not perfectly accurate, but we think it results in a decent estimation of the internal energy. Substitution into eq.(3), cancels the mass densities $\rho_1$ and $\rho_2$, which results in eq.(6).

## Alternative

More accurate would be to include changes in the density in the integral of eq.(5), as you mentioned. From the assumption that we have a constant bulk modulus $\bar{K}_s$, compressing a volume between state 1 and state 2 can be described as:

$$V_1 \frac{p_2 - p_1}{\bar{K}_s} = V_1 - V_2 \quad \rightarrow \quad V_2 = V_1 \left( 1 - \frac{p_2 - p_1}{\bar{K}_s} \right) \tag{b}$$

with $V_1$ the volume at pressure $p_1$, and $V_2$ the volume once $V_1$ is compressed to $p_2$. Using mass densities at the two states and the fact that we have a constant mass, we get:

$$\frac{m}{\rho_2} = \frac{m}{\rho_1} \left( 1 - \frac{p_2 - p_1}{\bar{K}_s} \right) \quad \rightarrow \quad \rho_2 = \rho_1 \left( \frac{\bar{K}_s}{\bar{K}_s - (p_2 - p_1)} \right) \tag{c}$$

Suppose that at state 0, the pressure $p_0$ = 0 bar, and the mass density equals $\rho_0$. We get the following function for the mass density at pressure *p*:

$$\rho(p) = \rho_0 \left( \frac{\bar{K}_s}{\bar{K}_s - p} \right) \tag{d}$$

The internal energy integral from state 0 to state $i$, as shown in eq.(a) above, now results in the following:

$$u_i - u_0 = \int_{p_0}^{p_i} \frac{p}{\rho \bar{K}_s} \mathrm{d}p = \int_{p_0}^{p_i} \frac{p \left( \bar{K}_s - p \right)}{\rho_0 \bar{K}_s^2} \mathrm{d}p = \frac{1}{\rho_0 \bar{K}_s^2} \int_{p_0}^{p_i} p \left( \bar{K}_s - p \right) \mathrm{d}p \tag{e}$$

Since we assumed $p_0$ = 0 bar, this results in:

$$u_i - u_0 = \frac{1}{\rho_0 \bar{K}_s^2} \left( \frac{p_i^2}{2} \bar{K}_s - \frac{p^3}{3} \right) = \frac{p_i^2}{2\bar{K}_s} \left( \frac{\bar{K}_s - \frac{2}{3} p_i}{\rho_0 \bar{K}_s} \right) = \frac{p_i^2}{2\rho^* \bar{K}_s} \tag{f}$$

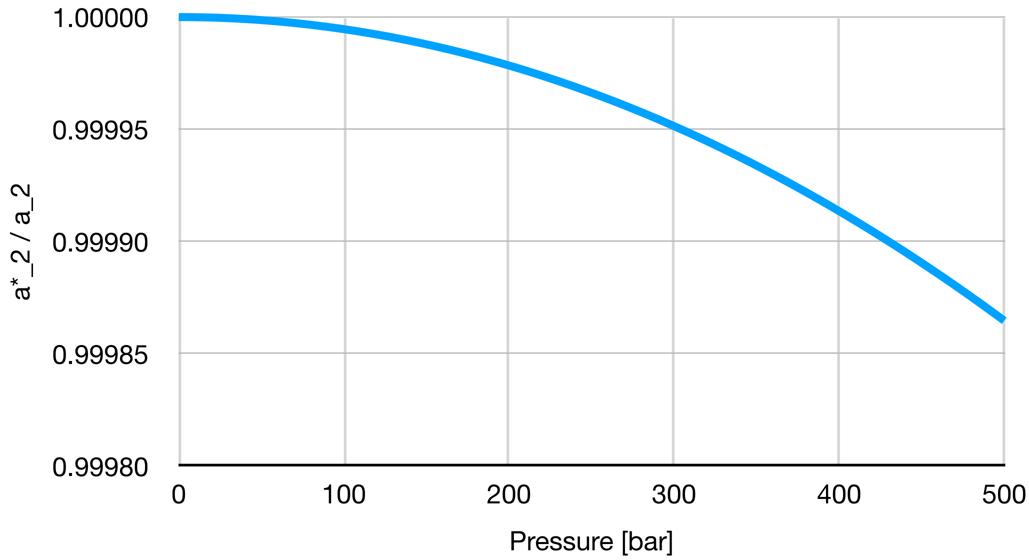
The function of $\rho^*$ shown in eq.(f), looks a lot like the function of $\rho$ we found in eq.(f). The difference is found to be:

$$\frac{\rho_i}{\rho_i^*} = \frac{\bar{K}_s - p_i}{\bar{K}_s - \frac{2}{3} p_i} \tag{g}$$

Following the same reasoning as in paper, this results in the following simplified function for the hydraulic power:

$$P_{hydr} = \left(1 + \frac{p_2}{2\bar{K}_s} \frac{\rho_2}{\rho_2^*}\right) p_2 Q_2 - p_1 Q_1 = a_2^* p_2 Q_2 - p_1 Q_1 \tag{h}$$

If we compare this factor $a_2^*$ to the $a_2$ factor set in the paper in eq.31, and we assume an isentropic bulk modulus of 1.76e9 Pa, we get difference of less than 0.015% at 500 bar, as is shown in the following graph.




In our opinion, this difference is so small that it justifies using mass densities $\rho_1$ and $\rho_2$, instead of more realistic functions.

6. Response by Peter Achten and Robin Mommers to the first rebuttal: 'note_210805_Response_Review_AchtenEtAl' (send on August 13, 2021 to the organizers of the FPMC and to the authors)

# Response to 'note_210805_Response_Review_AchtenEtAl'

We will only respond to the main points:

1. "Definitions are never wrong" (pag 1), "A definition is never wrong or impossible" (pag. 4)
2. We are (physically) not consistent in our approach for determining the overall efficiency.
3. We assume the density to be constant
4. Because of this inconsistency, we are not able to define a volumetric efficiency

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## Comments to point 1

In principle, these are correct statements: you can make any definition, as long as you are clear about the parameters and their meaning. For instance, you can define an efficiency ratio of the number of storks in a country and the number of babies born in a year. In 2019 there were about 2350 storks in the Netherlands. In that same year, 167.588 babies were born in the Netherlands. That is an amazing 'efficiency' of 71 babies per stork. Not that this is relevant, but yes, you could theoretically make such an efficiency definition. This efficiency is a 'defined quantity' and it is 'easy to apply, practical for users, and based on a transparent derivation'.

But that is of course not the point. We started this discussion because we believe the current ISO-definitions need a revision, resulting in better, physically consistent definitions. In our 2019-paper we write:

> "ISO 4409 is inconsistent in the calculation of the effects of oil compressibility: while it requires consideration for oil compressibility in the flow rates, it does not demand the same correction for the efficiency definition"

Because of this inconsistency, the current efficiency definitions result in the possibility that the efficiency can become larger than 1 (or 100%), which would imply that the pump or motor would have a negative loss. We believe we both agree that in that case, the definition is wrong, despite your remark that 'definitions are never wrong'.

Also in your comments, as well as in your paper, you often mention the need for (physical) consistency. According to you, our analysis is inconsistent i.e. wrong. Also here, the discussion is simply about good or wrong definitions, for which 'consistent' and 'inconsistent' are mere euphemisms.

Also in your paper, you write

> "However, the overall efficiency definition and the definitions of the partial efficiencies, namely the volumetric efficiency and mechanical-hydraulic efficiency are physically consistent only for an incompressible flow with the density $\varrho$=const. If the machine operates at high pressure levels the compressibility of the fluid and the dead volume of a pump must be taken into account. On this point, ISO 4391:1984 is physically inconsistent."

But that is only part of the problem. The other problem with the current ISO-definitions is that part of the energy which is compressed and delivered by the pump is actually leaving the pump in the form of a compressed oil flow. In motors, this compressed oil flow is received as an extra energy input. The pump and motor cycles are therefore not closed cycles, but they are open. More about that later.

## Comments to point 2

On page 2 you write:

> "Your derivation of the overall efficiency definition is inconsistent with the definition of the mechanical hydraulic efficiency. You integrate the inner energy (see your paper equation (5)) under the assumption of a mean density $\bar{\varrho}$ which is approximately $\bar{\varrho} = \varrho_1 = \varrho_2$ (see equation (6)). This assumption is neither transparently presented nor consistent with your assumption of a pressure dependent density (or volume) regarding the cycle of an ideal positive displacement machine (see your paper figure 2)). If your argument of a negligible error is made for equation (7) we do not understand why this should not also apply to figure 2 and equation (13) in your paper?"

However, as we have mentioned in our review, our definition of the overall efficiency is nearly identical to yours. To repeat our earlier remarks (we have renumbered the equations in order to make a new sequence for this document):

Furthermore your equation for the overall efficiency can be rewritten as follows:

$$\eta = \frac{\Delta p Q_2}{2\pi M_S}\frac{(1-\bar{\kappa}\Delta p/2)}{(1-\bar{\kappa}\Delta p)} = \frac{\Delta p Q_2}{2\pi M_S}\frac{(1+\bar{\kappa}\Delta p/2-\bar{\kappa}\Delta p)}{(1-\bar{\kappa}\Delta p)} = \\ = \frac{\Delta p Q_2}{2\pi M_S}\left(1+\frac{\bar{\kappa}\Delta p/2}{(1-\bar{\kappa}\Delta p)}\right) \tag{1}$$

This is rather similar to our equation:

$$\eta = \frac{p_2 Q_2 (1+\bar{\kappa}\Delta p/2) - p_1 Q_1}{2\pi M_S} \tag{2}$$

If, for the moment, we ignore the fact that we split $p_2$ and $p_1$, whereas you consider the pressure difference $\Delta p$, than our correction term is:

$$(1+\bar{\kappa}\Delta p/2) \tag{3}$$

whereas yours is:

$$\left(1+\frac{\bar{\kappa}\Delta p/2}{(1-\bar{\kappa}\Delta p)}\right) \tag{4}$$

Again using the parameters mentioned in Eq(6), our correction factor has a value of 1,0120 and yours of 1,0123, a difference of 0,0003.

We used the following values:

$$\begin{aligned} &\bar{\kappa} = 6\text{E}-10 \ [\text{Pa}^{-1}] \\ &\Delta p = 400 \ [\text{bar}] = 4E7 \ [\text{Pa}] \\ &V_d / V = 0.7 \ [\text{-}] \end{aligned} \tag{5}$$

In your current paper you need to substitute eq. 23 into eq.24 to get the same as the above equation (7). This shouldn't come as a surprise. We follow the same path as you do. We still disagree that you assume $\Delta pQ = p_2Q_2 - p_1Q_1$ (more about in our comments on point 4) but aside from this, the result is nearly identical. Which means that our, according to you ‘inconsistent’ approach results in the same value as your ‘consistent’ approach, which can be considered remarkable.

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## Comments to point 3

This seems to have become the core of your comment to our analysis. As you write in section 4 of your paper:

> “On the other hand, Achten et. al.’s definition of the overall efficiency is physically inconsistent. They integrate the inner energy (see [2] equation (5)) neglecting the pressure dependent density $\varrho$. Consequently, this leads to a physically inconsistent result of the hydraulic power as well which is the nominator of the overall efficiency.“

However, we believe you misunderstand our analysis at this point. As was also explained in more detail in the e-mail we sent on August 6th, when going from eq.5 to eq.6 we don’t use a constant mass density. We use eq.5 to estimate the amount of internal energy at a certain state . Since we need to compare the energy level to another state, we choose a state at which the energy is known to be very low (state 0 with $p_0$ = 0 bar). For simplicity, we indeed assume that the mass density in this integral is constant, but not to the mass density in state 0, but the mass density in state $i$:

$$u_i - u_0 = \frac{1}{\rho_i \bar{K}_s} \int_{p_0}^{p_i} p\,\mathrm{d}p \quad \rightarrow \quad u_i = \frac{p_i^2}{2\rho_i \bar{K}_s} \tag{6}$$

This indeed is not perfectly accurate, but we think it is a decent estimation of the internal energy. If we implement this into eq.3 of our 2019-paper, the mass densities cancel out, which results in eq.6. More accurate would be to include changes in the density in the integral of eq.5 as you mentioned, but that would only result in a very small change of the end result, which we think can be neglected. It should also be noted that you do the same in your analysis.

You could have seen from our equations that we are not assuming the density to be constant. Otherwise we wouldn’t need the bulk modulus and we couldn’t have an isentropic compression and expansion in the ideal cycle. Like you do, in eqs.7 and 8 in your paper, we assume the relation between volume $V$ or density $\varrho$ and pressure $p$ of a fluid to be described by the bulk modulus, which implies that the density is by definition variable. This still leaves room for differences in the choice of the reference volume ($V_{\mathrm{eff}}$ in your case versus $V$ in our paper), but that doesn’t change the end result, as has been made clear before.

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## Comments to point 4

This is another important point in your paper. At the end of your paper, in section 4, you write

> “A physically consistent volumetric efficiency definition that fulfils $\eta = \eta_{\mathrm{mh}}\,\eta_{\mathrm{vol}}$ and that is based on the idea of an ideal and reference machine is not achievable due to their inconsistent integration of the inner energy. In summary, they apply the idea of an ideal machine in the context of their mechanical-hydraulic efficiency definition, but and in contrast to this paper not to the volumetric efficiency definition and volumetric losses.”

However, in our 2019-paper we write:

“It should be noted, that the definition for the volumetric efficiency is not based on energy or power levels, but compares flow rates. In order to calculate a volumetric efficiency based on power or energy, both the measured and the theoretical flow could be multiplied with a pressure, but then the question is which pressure level should be used for the numerator and for the denominator. Any choice would be an arbitrary choice, and may question the validity of the definitions.”

Furthermore, in our review of you paper, we write:

“We didn’t provide a definition of the volumetric efficiency because we couldn’t find a definition which was physically consistent with the inner processes in hydrostatic pumps and motors. You can make and define a flow ratio, but that is not the same as a power or energy ratio (which was the topic of our paper).

The fact that we didn’t come up with a definition of the volumetric efficiency has nothing to do with our definition of the overall efficiency. After all, in theory, it could be possible to make an equation in which our overall efficiency is divided by our definition of the hydro-mechanical efficiency, which would then result in a ‘volumetric efficiency’ which fulfils $\eta = \eta_{mh}\ \eta_{vol}$. But this would not make any sense due to the reasons mentioned before.”

Trying to come to a consensus in this discussion, we listen and read your comments with great care. We believe it would help us both, if you would carefully read the above statements and respond to them.

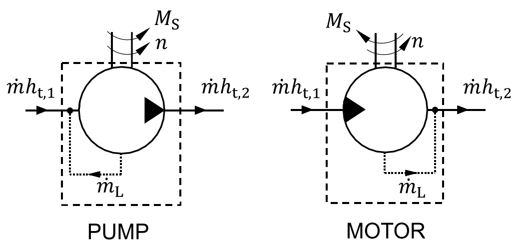
We believe we have reached a consensus about the hydraulic-mechanical losses and efficiency definitions. As you also mention in your response to our review:

“the differences for mechanical hydraulic efficiency representation are negligible."

We also believe we have reached a consensus about the calculation of the overall losses and efficiency definitions, although you still seem to rescind this. But, as mentioned before, we can’t see any fundamental differences between your analysis and ours. The differences in the numerical end results are negligible and are due to different points of linearisation.

However, we believe you are making a mistake when you believe that all volumetric losses originate from the same $\Delta p$: the pressure difference between the high pressure side and the low pressure side of the pump or motor.

In your paper (Fig. 1) you consider the same mass flow and the entrance and exit:




In order to fulfil this balance you assume that all volumetric losses inside the pump or motor will eventually end up at the low pressure side of the pump. In reality, this means that you need a pressurised case or housing, otherwise, the internal leakage will not flow to the low pressure side.

However, then you neglect the reality that many pumps and motors don't have an internal case drain, but instead an external case drain. And also that in many applications, for instance in all hydrostatic drives, the low pressure is substantially higher than the case pressure. In these pumps, the leakage in the housing must be directed to the external drain.

As a result, it is no longer true that the mass flow at the exit equals the mass flow at the entrance, as you show in Figure 1 of your paper. In your comments on our review you write:

> "Your argumentation is from a standpoint of an application. The efficiency definition is from a standpoint of a standardized procedure under standard conditions. A higher pressure at the inlet is possible, the question is whether this must be considered for the standardized efficiency measurements. Nevertheless, we see no limitation for our consideration."

The answer to your question is confirmative: yes, of course you need to consider how the pumps and motors are applied. The standards should reflect the reality, not the other way around. However, this is what you do when you write:

> "In our consideration $\dot{m}_1$ und $\dot{m}_2$ are equal due to our chosen system boundary (see Figure 1). We do not see a benefit in your suggested equation. In addition, the challenge becomes apparent when $\dot{m}_1$ und $\dot{m}_2$ are not equal."

You want your definitions to be:

> "…valid for all pumps and motors, also with external drainage, as can be seen in our figure 1 and our chosen system boundary"

It should be clear by now that this is not possible, following your analysis.

An other consequence of the external leakage drain in many pumps and motors, is that the leakage can come from both the high pressure side and the low pressure side. This is especially relevant for closed circuit pump and motors, in which the low pressure is around 20 bar, and the case pressure around 0 bar. Please, explain to us how you can tell where the measured leakage is coming from? We can't, which was one of the reasons why we decided not to make a volumetric efficiency definition.

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## Conclusion

Considering the discrepancies and the lack of sufficient answers and responses to our first review, we believe we need more time to finalise our discussion. We will be more than happy to continue the debate and are open for any additional comments from your side.

The paper in its current form does not deserve the mark of a peer reviewed paper, at least not having us as reviewers. **We will therefore inform the organisers of the FPMC2021, and advice them to withdraw your contribution from the 2021 conference.**

We have reached consensus about several points:

1. We both agree that there is need for a revision of the current efficiency definitions for pumps and motors, as defined in ISO-standards;

2. We both come to about the same definition for the hydraulic-mechanical efficiency. The differences are negligible.

Then there is a point where we (the reviewers) are certain that we reached a consensus, but you completely and strongly disagree:

3. We (the reviewers) also believe that we have reached the same level of consensus about the overall efficiency and loss definitions. We regret that you don't share this conclusion in your last paper. We also regret that you didn't respond to the content of our numerical example as mentioned in our review (which is repeated again in this second review (see the comments to point 2).

Finally we continue to have a disagreement about the definition of the volumetric efficiency.

4. Contrary to your statements we decided not make a definition of the volumetric efficiency because there is not a method to assess the pressure level from which the leakage originates. You consider all leakage to originate from the high pressure side. That is not the reality for a large group of pumps and motors.

August 13, 2021
Peter Achten and Robin Mommers (INNAS)

PS: One last word about point (iii) in the introduction of your paper. If you are looking for a Carnot-like maximum boundary of the efficiency of a pump or motor, then the answer is simply 100%. Like electric motors or gear transmissions, the transformation in hydraulic pumps and motors does not, at least in principle, involve any loss of entropy. We have already measured values up to 98%, so we are getting close. Although we are also realistic that we won't achieve 100%, but neither does an electric machine or a gear transmission.

7. Response from Christian Schänzle and Peter Pelz to the previous document Note_210818_Response_Review_Achten_Pelz_schaenzle (received on August 21, 2021)

Dear Dr Peter Achten, Dear Robin Mommers,

In the following, you find our response to your review that we received on August 13th, 2021. At the beginning, we want to make a statement considering the review process so far and, secondly, give a short summary of the consensus and differences.

<u>Statements to review process</u>

On June 15 we got the notification from ASME that the review process for our paper was completed and that our paper was accepted. Two reviewers were very positive, one reviewer was neutral and you were very critical. This might be because we noted in your paper weak points regarding the three "c", i.e. conciseness, consistency and clearness, cf. Heinrich Hertz's book "Mechanics" and also Occam's razor. We further noticed in the review process a different understanding what are physical axioms, definitions, models. From an engineering point of view, we missed the difference of function and quality and the different importance of function and quality for the various stakeholders, i.e. OEM, owner-operator, manufacturer, society, science.

For us the most severe point is the following: You try to convince the community that the volumetric efficiency should be banned (once again this word). There are three reasons why the volumetric efficiency is of value for different stakeholders:

1. OEM, Owner-Operator: Separating function and quality (energetic quality, ….) as an engineer should do, the volumetric efficiency gives the pump or motor function or characteristic. There are many kinds of positive displacement pumps. To give you an example: For screw pumps the function is a required volume flow and the most important characteristics for the customer is the volumetric efficiency of the pump.
2. Manufacturer: Modelling the total efficiency of a machine requires the separation of volumetric losses and internal pressure losses. Hence, for the modelling engineer the separation in mechanical hydraulic and volumetric efficiency is beneficial.
3. Scientist: It is indeed possible to show that the volumetric efficiency is an energetic measure as we did in our paper which is of interest from a purely scientific point of view. But also the task of modeling the overall efficiency it is beneficial to focus on volumetric losses and pressure losses separately.

Looking at all stakeholders in the community the first point is obviously the most important one. There should be very clear, objective, convincing and consistent reasons and arguments to tell the customer that the volumetric efficiency is outdated. Your argumentation in your paper and in this dialog is from a scientific point of view too weak to support this.

Both you and we put a lot of input and effort into the open review process for the sake of the community as well as for our own's sake. Once again, we thank you for your comments.

In our understanding we did all that was expected from us and where we agree to the suggestions for improvement. In fact, there are no errors in our paper as you suggested in your last email. This was confirmed by the three other reviews. So, we disagree with you in some points and that is perfectly normal and the full right of authors. In no way was the review a request to conduct an intense review process with you as also stated by Perry Li in his email on 16 August. Of course, we acknowledge the fruitful discussion so far.

In our presentation at the FPMC 2021, we will address the consensus and the differences stated below in a fair, respect full and open way. We will make clear three paper exist that critically examine the efficiency definition for positive displacement machines:

> [1] Achten et al.: “Measuring the losses of hydrostatic pumps and motors: A critical review of Iso4409:2007”, FPMC 2019
>
> [2] Li and Barkei: “Hydraulic effort and the efficiencies of pump and motors with compressible fluid”, FPMC 2020
>
> [3] Schänzle and Pelz: “Meaningful and physically consistent efficiency definition for positive displacement pumps - continuation of the critical review of iso 4391 and iso 4409”, FPMC 2021

All three paper [1] to [3] are in a nice historical row taking up a discussion. The Fluid Power Community is critical and thoughtful. Thus, the community is able to make up its own mind about the content and the scientific discussion based on the three papers mentioned above. Against this background, a complete consensus between us is not necessary and may be not achievable. Instead, integrating the community in our ongoing discussion is the next logical step.

To make our discussion public to the community we suggest to publish our discussion to date (e.g. via TUBiblio our University Library with DOI). Hence, the open dialog, initiated by you, would be open to the community as well. Please answer us, if you agree on this. We will make our letters public to the Fluid Power Community at least to the German Fluid Power Community.

<u>Consensus and differences</u>

Consensus

- Difference in representation of mechanical hydraulic efficiency leads to neglectable differences in mechanical hydraulic efficiency values
- Difference in representation of overall efficiency leads to neglectable differences in overall efficiency

Differences

- Consistency of approach deriving partial efficiencies
- Understanding of volumetric efficiency and its relevance for the manufacturers

Peter Pelz and Christian Schänzle

PS: Response to your second review on 13th August, 2021

**Response to your second review on 13th August, 2021**

**Response to comments to point 1:**

We want to underline our statement, that “definitions are never wrong”. Thus, consistency is not a euphemism for “wrong”.

**Response to comments to point 2 and 3:**

We follow your argumentation that values obtained by our different efficiency definitions are nearly identical. However, in our opinion, this makes your approach defining the overall and the mechanical hydraulic efficiency independently of each other not consistent.

The way of integration of the inner energy in your paper is neither transparent nor comprehensible. Not including the density in the integral and the associated assumptions and simplifications as presented in Mr. Mommers' email on 6th August are not available to the reader. In our opinion, our misunderstanding of your approach is not our fault, but a result of the non-transparent assumptions.

At the same time, we are not able to apply this form of integration of the inner energy (equation 5 in your paper) on the compression of $V_{\max}$ (see Figure 2 in your paper) obtaining the term $\frac{\frac{1}{2}\Delta p^2 V_{max}}{\overline{K}_S}$:

$$\frac{\frac{1}{2}\Delta p^2 V_{max}}{\overline{K}_S} \neq \Delta U - \frac{p_1 V_{max} \Delta p}{\overline{K}_S}$$

$$\frac{V_{max}}{2\overline{K}_S}(p_2^2 - 2p_1p_2 + p_1^2) \neq m\left(\frac{p_2^2}{2\varrho_2\overline{K}_S} - \frac{p_1^2}{2\varrho_1\overline{K}_S}\right) - \frac{p_1 V_{max}\Delta p}{\overline{K}_S} \qquad |\varrho_1 = \frac{m}{V_{max}}, \varrho_2 = \frac{m}{V_2}, V_2 = V_{max}\left(1 - \frac{\Delta p}{\overline{K}_S}\right)$$

$$\frac{V_{max}}{2\overline{K}_S}(p_2^2 - 2p_1p_2 + p_1^2) \neq \frac{p_2^2 V_2}{2\overline{K}_S} - \frac{p_1^2 V_{max}}{2\overline{K}_S} - \frac{p_1 V_{max}\Delta p}{\overline{K}_S}$$

$$\frac{V_{max}}{2\overline{K}_S}(p_2^2 - 2p_1p_2 + p_1^2) \neq \frac{V_{max}}{2\overline{K}_S}\left(p_2^2 \frac{V_2}{V_{max}} - 2p_1p_2 + p_1^2\right)$$

In our opinion, this makes your approach inconsistent. This does not mean your approach is wrong, but your assumptions are not consistent. This leads to representations, which in our opinion are unnecessarily complicated.

**Response to comments to point 4:**

We presented our view on the volumetric efficiency in our last review and in our revised paper. Therefore, we will not repeat it.

In addition to the possibility of using the volumetric efficiency as a ratio of two energetic quantities, the volumetric efficiency also represents an essential quantity describing the function of a machine, whereas the overall efficiency measures the energetic quality of a machine. In our understanding, a separation of the function and the quality is essential for a sustainable systems design. Moreover, the customer of a pump manufacturer is mostly interested in the function of a machine described by the volumetric efficiency. Your arguments for no longer naming the volumetric efficiency do not convince us. Neither from the customer's point of view nor from the manufacturer's point of view.

We accept the fact that for your mentioned application, e.g. a closed circuit pump with a low pressure around 20 bar, our system boundary and circuitry (see Figure 1 in our paper) is not applicable. Nevertheless, we are transparent with our system boundary and present our assumptions in a transparent and comprehensible way. Furthermore, your argument is that the obtained values from the different definitions are nearly identical.

8. Comment from Robin Mommers and Peter Achten to Note_210818 (Comment_210825_note_Pelz, send to Christian Schänzle and Peter Pelz on August 25, 2021)

Dear mr Pelz, dear mr Schänzle,

In this comment, I will not respond to the discussion about the volumetric efficiency. We'll be happy to discuss this at a later point. I do want to make a comment on some of the choices that we made in our paper, and on the inequality you shared with us in you previous note.

In your response, you mention several stakeholders in the hydraulic industry:

1. OEM, Owner-Operator
2. Manufacturer
3. Scientist

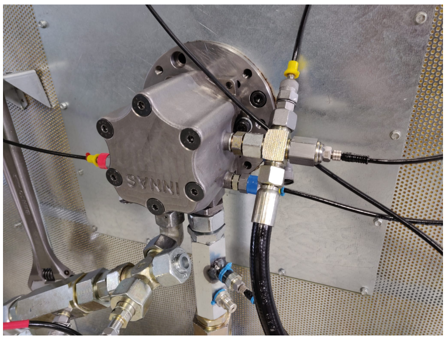
I think it is fair to say that your perspective is mainly from the third group, while we might be more between the second and the third group. If one looks at any definition (in this case "the way to measure efficiency"), we look at this from our own perspective. I mention this, because I think that this might explain some of the differences in the assumptions that both of us have made.

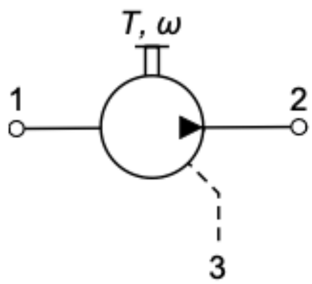
## Our perspective

This morning I have been measuring the performance of one of our prototype pumps, which you see in this photo below. This machine has a supply line (right side on bottom), a discharge line (left side on bottom), and a drain line (hose coming out at the side). This is not unique, and is actually the case for most pumps that we test on our test bench. In accordance with ISO4409 standard, we measure:

- the pressure and temperature of all lines
- flow rate of discharge and drain line
- torque and speed of the axle (actuated on the other side of wall)

Oil that leaves the pump via the drain port is directed to a tank (0 bar). A smaller pump pumps oil from the tank into the low pressure circuit (often between 5 and 15 bar), from which the tested pump is fed.






The symbolic representation of this pump is shown on the right, which you might recognise as figure 1 from our paper. Since the left figure is our starting point, and we want to include all possible pumps, we chose our system boundaries different than you did (note that when there is no external drain port, our derived system can still be used).

In one of your earlier comments you mentioned that choosing the system boundaries like this will mean that you no longer have a constant mass flow. We agree, and this is one of the challenges we had in the writing of our paper. However, as mentioned above, we tried to find a relation that is applicable to all pump and motors, and not just machines without an external drain. In our experience, there are a lot of displacement machine that have an external drain port.

In recent years, there have been some units for which a mechanical efficiency was measuring of more than one. As a result, it was becoming clear that the effects of compression can no longer be ignored when calculating the efficiencies of pumps and motors. This is why we started working on better definitions, which is still an ongoing discussion, as you are well aware.

## Our paper

In the first part of our paper, we are only looking at the flow of energy and thus power in the machine. Using the thermodynamic system with the boundaries shown in the symbolic pump representation above, and some common assumptions (e.g. no power loss due to radiation), we form some basic equations to determine the power loss. The mechanical input power ($T\omega$) is converted to hydraulic power (which is the output power), and some of this converted power is lost via the drain line. The amount of power loss for this pump is thus found to be:

$$P_{loss} = P_{in} - P_{out} = T\omega - P_{hyd} \tag{1}$$

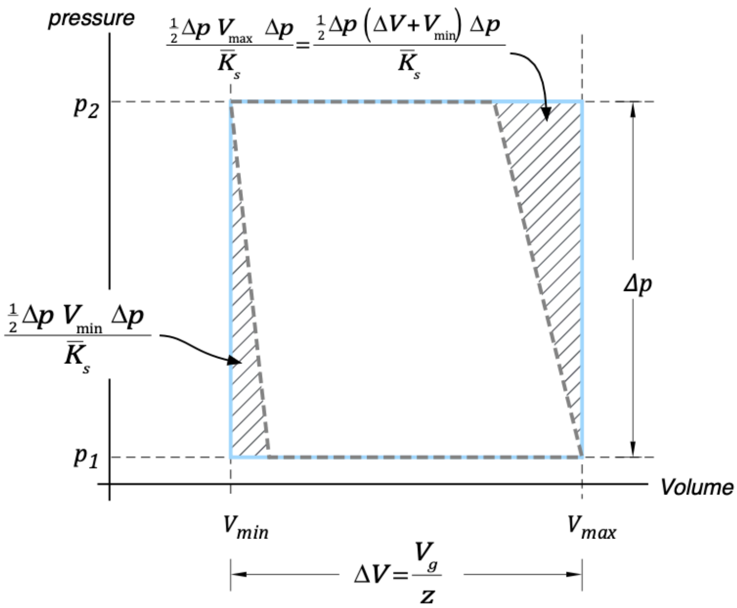
We both agree on this definition, but we have different opinions about the loss that occurs via the drain line.

In the second part of our paper, we are trying to determine what is the ideal torque. The pV-diagram is a very powerful tool for understanding why the compression effects should be taken into account, which is probably why you also have several pV-diagrams in your paper. In your last response you show that there is an inconsistency in the way we calculate the compression energy during the integration of the internal energy and when we are interpreting the pV-diagram. While this is factually correct, this is merely an artefact of using a constant bulk modulus and only using the first term of the Taylor expansion that is related to the volume change.

Allow me to explain. You wrote the following:

$$\frac{\frac{1}{2}\Delta p^2 V_{max}}{\bar{K}_s} \neq \Delta U - \frac{p_1 V_{max} \Delta p}{\bar{K}_s} \tag{2}$$

The left side of the inequality sign comes from figure 2 of our paper, which is the same as the pV-diagram shown below. The right side comes from the integral that we defined earlier (equations 5 and 6). If I understand correctly, you are calculating the energy change when you start at pressure $p_1$, volume $V_{max}$, and compress to pressure $p_2$, volume $V_2$.



The figure is showing the the dashed area is almost a triangle, which means that the size of the area will be very close to $0.5 \cdot \Delta p(V_{max} - V_2)$. The change in volume that is between brackets is also part of the second term on the right side of the inequality sign in (2). So we need to determine the amount of volume change during commutation. The volume at $p_2$ due to compression is as follows:

$$V_2 = V_{max} e^{-\int_1^2 \frac{dp}{K}} \tag{3}$$

If we assume a constant bulk modulus, this simplifies to

$$V_2 = V_{max} e^{-\frac{p_2 - p_1}{K}} \tag{4}$$

Using only the first term of the Taylor expansion of the exponential function, the volume change is found to be:

$$V_2 = V_{max} \left(1 - \frac{p_2 - p_1}{K}\right) \quad \rightarrow \quad V_{max} - V_2 = V_{max} \frac{p_2 - p_1}{K} \tag{5}$$

However, since the integral that is in the exponential of (3) is reversible, we can just as well state that the following is true:

$$V_{max} = V_2 e^{-\int_2^1 \frac{dp}{K}} \tag{6}$$

Following the same reasoning, this results in:

$$V_{max} = V_2 \left(1 + \frac{p_2 - p_1}{K}\right) \quad \rightarrow \quad V_{max} - V_2 = V_2 \frac{p_2 - p_1}{K} \tag{7}$$

Both (5) and (7) describe the same amount of volume, but merely differ due to an arbitrary choice. This is similar to the conclusion you made for yourself at the end of section 3.1 of your paper, for which you find the difference to be negligible. We can consider this to be an “inconsistent” use of choosing (5) or (7), but as far as simplifications go, they are effectively “the same”. As you also mentioned, we wanted to focus on what can be used practically. We believe that our way of formulating it is the most practical one. While the equations you found are slightly different, they are almost identical.

## Actual discussion

This leaves me to address an actual point of discussion, which you might have missed in our first review. We asked you how you “How do you suggest that $V_{eff,1}$ can be measured?”, since it is such an important parameter in the equations that you end up with. Your answer was that is can be calculated using equations (13) and (14) of your paper. However, the reason we asked this question was not because it was unclear how you calculate it.

The actual question was “How do you suggest that $V_{eff,1}$ can be measured, in practice?”. The dead volume of a working chamber is defined a $V_d$ in your paper, and as $V_{min}$ in ours. We asked this question, since we have not been able to find a good way to determine this dead volume in a commercial machine, other than looking at drawings (which are often drastically simplified for cleanliness) and measuring it with callipers and other measuring devices in the actual machine (which is very inaccurate). Do you have any ideas on this?